Prognostics based on the Fokker-Planck Equation

Reliability and RUL from Physical Models

Thomas Christen, Hitachi Energy Research, Baden-Dättwil, Switzerland

IMC 2023, Lausanne

2023-08-29

Hitachi Energy

This Presentation

- Motivation
  - Prognostics: important part of predictive (smart) maintenance
  - Easy-to-use (commercial) Multiphysics simulation tools for nonlinear partial differential equations exist

- Content
  - Modeling aging/degradation with stochastic differential equations and the Fokker-Planck equation
  - Reliability and Remaining Useful Lifetime (RUL)
  - Correction steps – update with new information (e.g., from diagnostics)
  - Examples and applicability extensions
    - Brownian motion with drift
    - Auxiliary variables
    - Variable failure boundaries
    - Chance failures in addition to wear-out failures

Ref.: T. C. and F. Macedo, “Theory of Fokker-Planck Equations for Reliability and Remaining Useful Lifetime Prognostics”, accepted by IEEE Transactions on Reliability, 2023

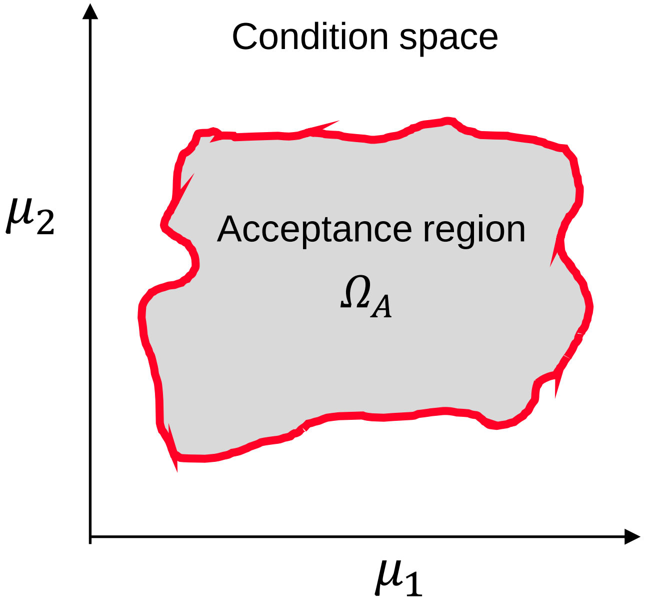
Device Model Variables and Parameters, and Condition Space

HITACHI
Inspire the Next

- Time dependent state variable $\vec{\phi}$ ∈ State Space (continuous time t)
- Device model parameters $\vec{\mu}^{(D)}$ ∈ Parameter Space Ω (Condition Space)
- Device operations represented by solutions $\vec{\phi}_{\vec{\mu}^{(D)}}(t)$ of dynamical system

$$\frac{d\vec{\phi}}{dt} = \vec{F}_{\phi}\left(\vec{\phi}, \vec{\mu}^{(D)}, t\right)$$

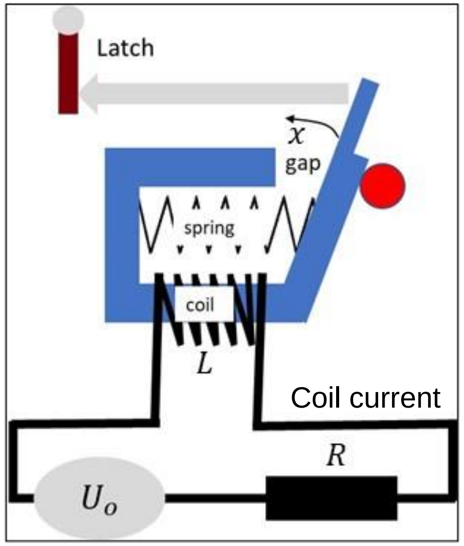
- Malfunction/failure ≡ inacceptable behavior $\vec{\phi}_{\vec{\mu}^{(D)}}(t)$
  - If $\vec{\mu}^{(D)} \in \Omega_A$ (Acceptance region) → device can perform its task
  - If $\vec{\mu}^{(D)} \notin \Omega_A$ → device fails to perform its task



Hitachi Energy

Examples

Electro-magneto-mechanical actuator

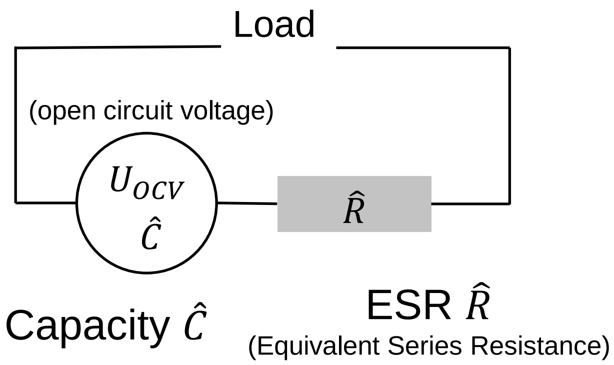


$$\vec{\phi} = \begin{pmatrix} x \\ I \end{pmatrix} = \begin{pmatrix} \text{mechanical coordinate} \\ \text{coil current} \end{pmatrix}$$

$\mu^{(D)}$ = required inception force (spring force, static friction, …)

Malfunction: unable to act with sufficient force and within sufficient time

Battery (other energy storage devices analogous)



$\phi = Q$ (state of charge, SOH)

$\vec{\mu}^{(D)} = \begin{pmatrix} \hat{C} \\ \hat{R} \end{pmatrix}$ - or normalized wrt beginning of life: $\begin{pmatrix} \hat{C}/\hat{C}_{BOL} \\ \hat{R}_{BOL}/\hat{R} \end{pmatrix}$

Malfunction: unable to provide sufficient $\begin{pmatrix} \text{energy} \\ \text{power} \end{pmatrix}$

Degradation and End of Life

HITACHI
Inspire the Next

- Beginning of Life (BOL): initial state $\vec{\mu}(0) = \vec{\mu}_{BOL}$ of aging model variables
- Aging or degradation: (slow) motion $\vec{\mu}(t)$ in acceptance region
  - Deterministic part ("drift", $\vec{f}$)
  - Stochastic part ("random walk", white noise $\vec{\xi}(t\,)$)

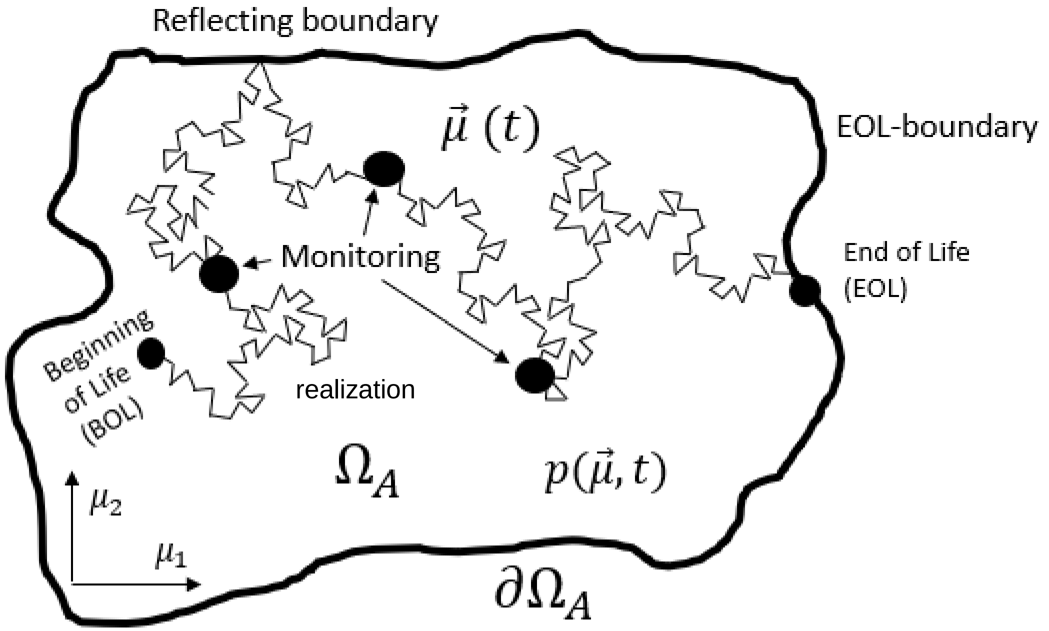
$$\frac{d\vec{\mu}}{dt} = \vec{f}(\vec{\mu}, t; \vec{\alpha}) + \boldsymbol{g}(\vec{\mu}; \vec{\alpha})\vec{\xi}(t)$$

(aging model parameters)

(Langevin equation, stochastic differential equation)

(note: aging variable space is generally larger than device model parameter space!)

- When boundary $\partial\Omega_A$ of acceptance region is reached:
  - Impermeable: reflecting boundaries
  - Malfunctions, End of Life (EOL): absorbing boundaries
  - …

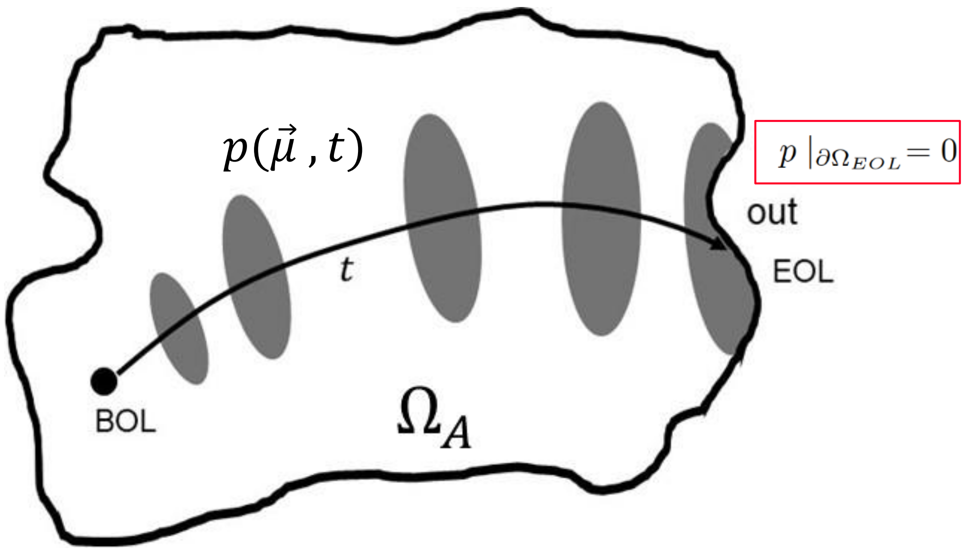


Hitachi Energy

Langevin Equation (LE) and Fokker-Planck Equation (FPE)

- FPE for probability density $p(\vec{\mu}\,,t)$ (individual device: "Bayesian interpretation")

$$\frac{\partial p}{\partial t} = -\nabla_\mu \cdot \vec{j}_\mu = \sum_i \frac{\partial}{\partial \mu_i}\left(\sum_j \frac{\partial}{\partial \mu_j}(D_{ij}p) - f_i p\right)$$

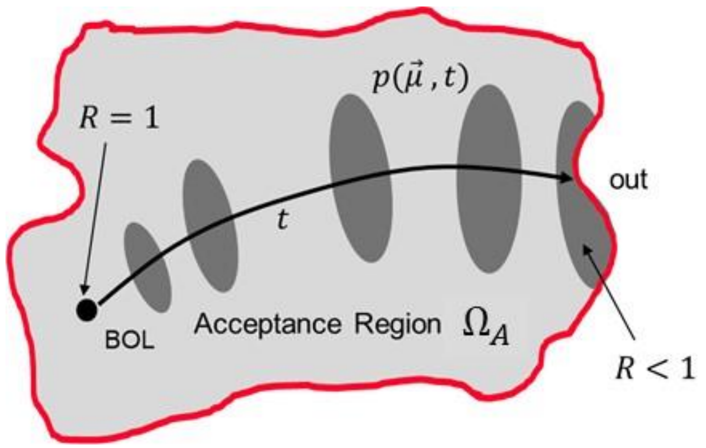


- FPE "equivalent" to LE ("diffusion ↔ Brownian motion")
  - Diffusion matrix $(D_{ij} = D_{ij}(\boldsymbol{g})$; ⚠Itô/Stratonovich-dilemma⚠)
  - Drift-flow velocity $\vec{f}(\vec{\mu},t)$ (can be arbitrarily nonlinear)
- Initial condition at BOL
  - $p(\vec{\mu}\,,t=0) = p_{BOL}(\vec{\mu})$ (usually) normalized in Ω_A
- Boundary Conditions
  - Reflecting/insulating boundary: vanishing normal component of $\vec{j}_\mu$ at boundary $\partial\Omega_A$
  - Absorbing EOL boundary: $p = 0$ at $\partial\Omega_A$
- Distribution (remains) generally not Gaussian (boundary conditions, nonlinearities, …)

Reliability and Remaining Useful Life

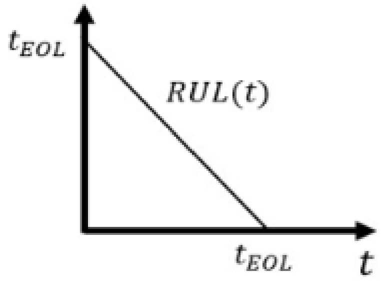
- Reliability from $p(\vec{\mu}, t)$
  - $R(t)$ = survival probability = probability to be in Ω_A

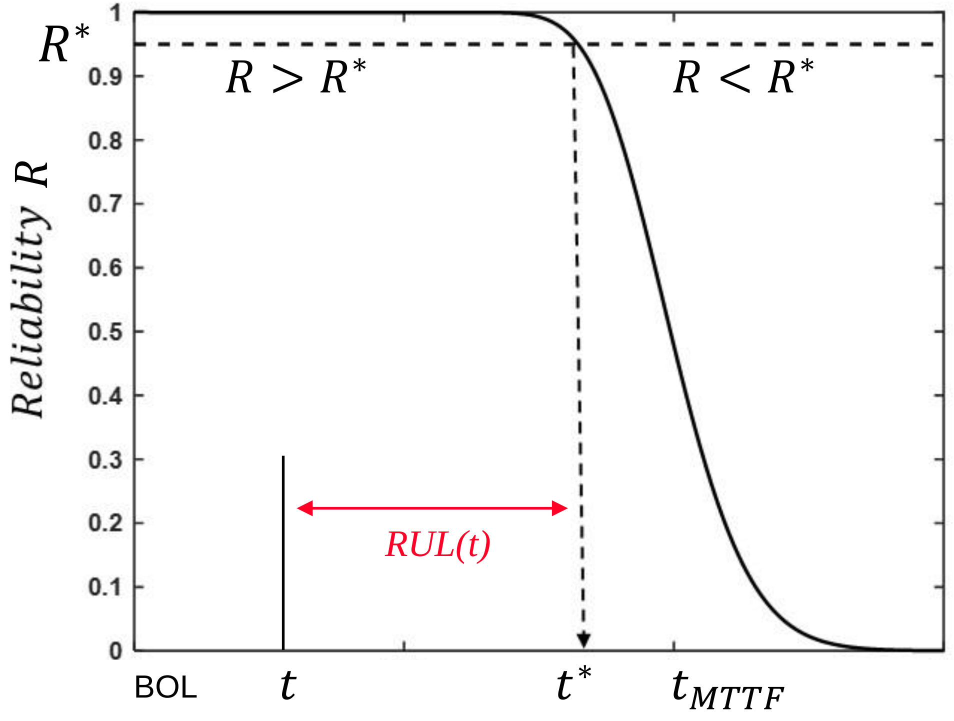
$$R(t) = \int_{\Omega_A} d\mu \; p(\vec{\mu}, t)$$



- Remaining Useful Lifetime (RUL)
  - Reliability limit R^*: device condition acceptable iff $R > R^*$
  - RUL = time until $R = R^*$

$$RUL(t) = t_{EOL} - t$$





Public

Intermediate Information Gain: Correction Step

- Up to now: <u>Prediction step:</u> $p(\vec{\mu},t)$ calculated from FPE until (discrete) time t_m → predicted condition probability at t_m

$$p_<(\vec{\mu},t_m) = \lim_{\epsilon\to 0\ ,\epsilon>0} p(\vec{\mu},t_m-\epsilon)$$

- Additional information gain (monitoring, diagnostics) at $t = t_m$ on $\vec{\mu}$ → likelihood

$$p_m(\vec{\mu},t_m)$$

- <u>Correction step</u>
  Update p with Bayesian inference

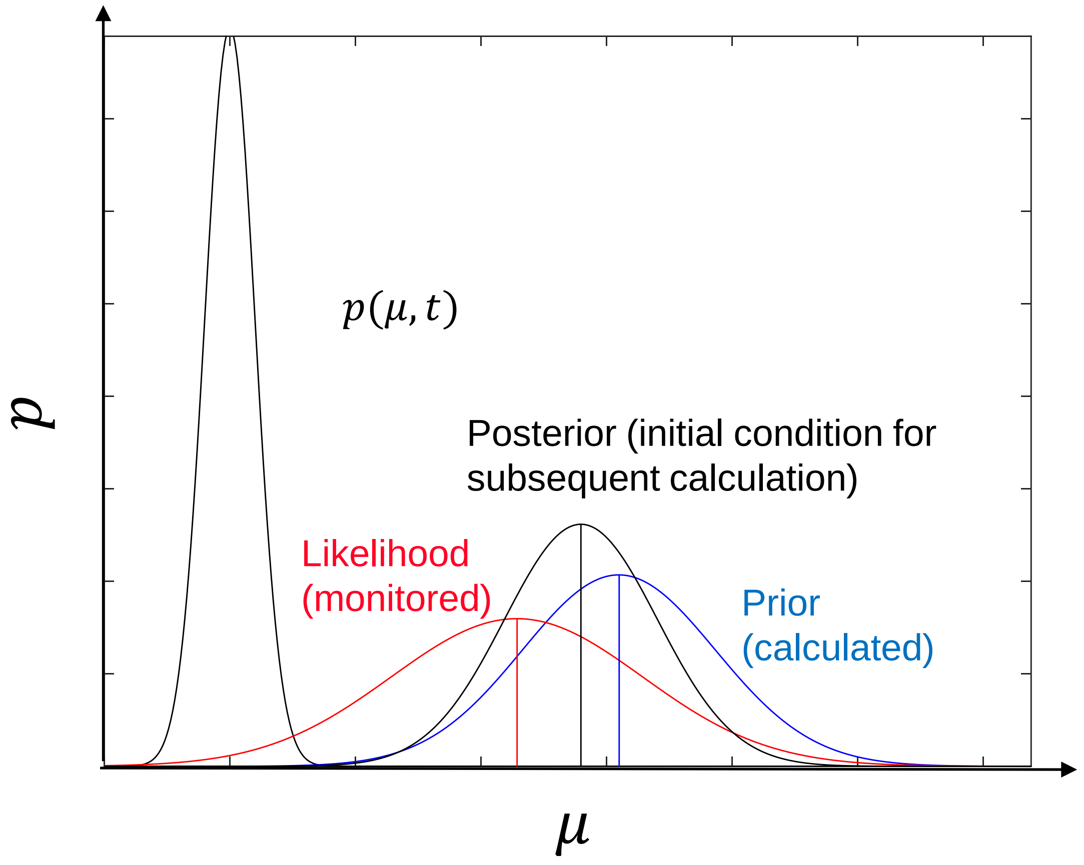
Posterior "Likelihood" prior

$$p_>(\vec{\mu},t_m) = C\ p_m(\vec{\mu},t_m)p_<(\vec{\mu},t_m)$$

Normalization constant C

- Continue to solve the FPE with "new initial condition"

$$p(\vec{\mu},t\,)|_{t=t_m} = p_>(\vec{\mu},t_m)$$



Summary for 1d Condition Variable Example, known $\vec{\alpha} = (f, D)$

HITACHI
Inspire the Next

- Aging model (Brownian motion with drift)
  Langevin equation ($\mu \geq 0$)

$$\frac{d\mu}{dt} = f + \sqrt{2D}\xi(t)$$

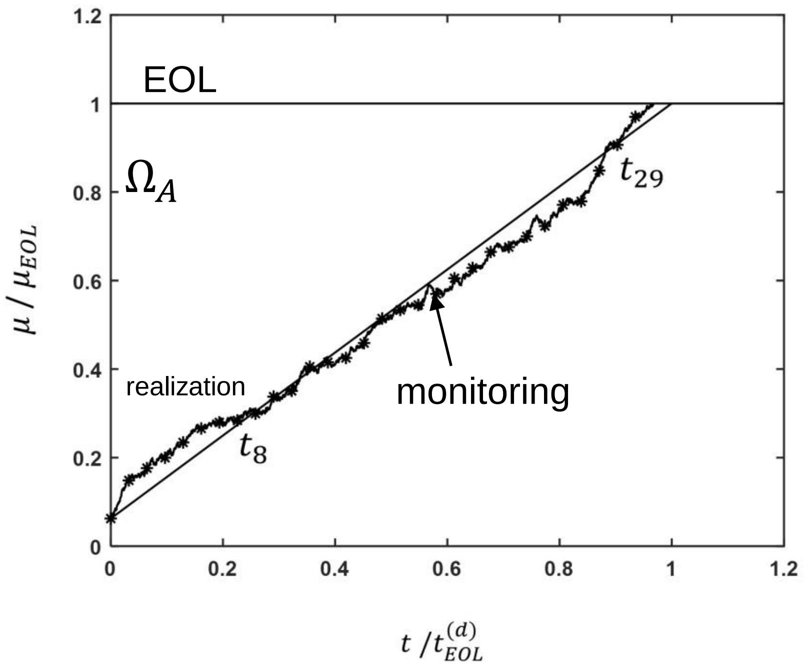
- Fokker-Planck equation (drift-diffusion)

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial \mu}\left(D\frac{\partial p}{\partial \mu} - fp\right)$$

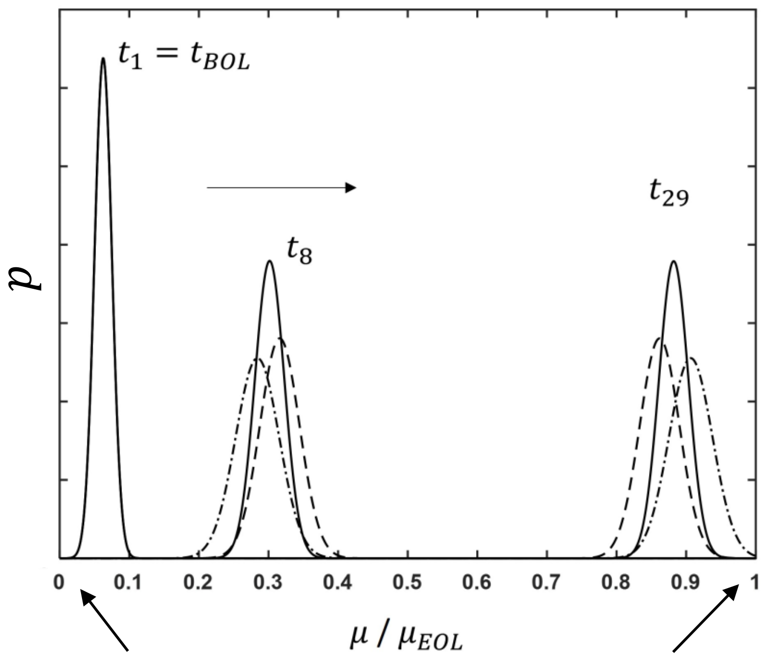
- Reliability and Lifetime

$$R(t) = \int_0^{\mu_{EOL}} d\mu\, p(\vec{\mu}, t)$$

Here: simulation of stochastic differential eq.



Simulation of partial differential equation



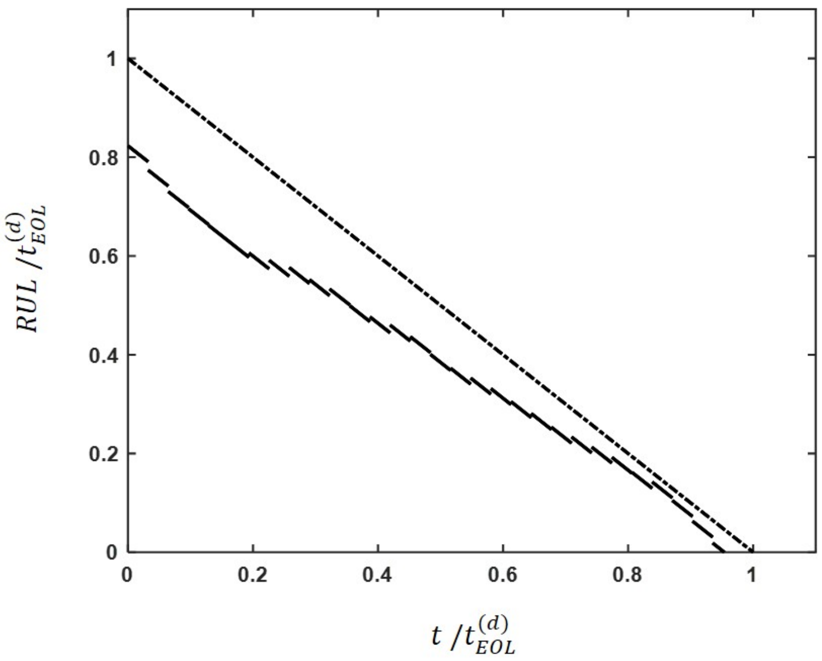
$(D\partial p/\partial\mu - fp)\big|_{\mu=0} = 0$

reflecting/insulating

$p\big|_{\mu_{EOL}} = 0$

absorbing

Calculation of RUL from reliability



(d) = deterministic

Hitachi Energy

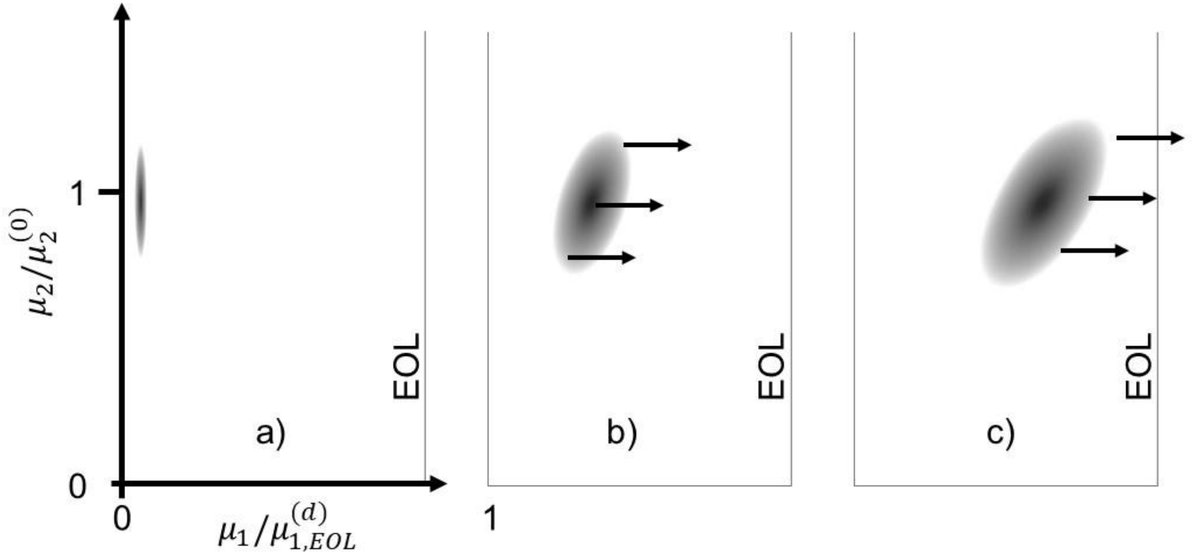
Auxiliary Aging Variables

What if some of the aging model parameter values $\vec{\alpha}$ are not known or variable? → Consider as additional stochastic variables

- Simple example: rate f of previous model is stochastic variable
  - f extension of aging model variable space

$$\mu_2 = f$$

FEM Simulation (here Comsol)



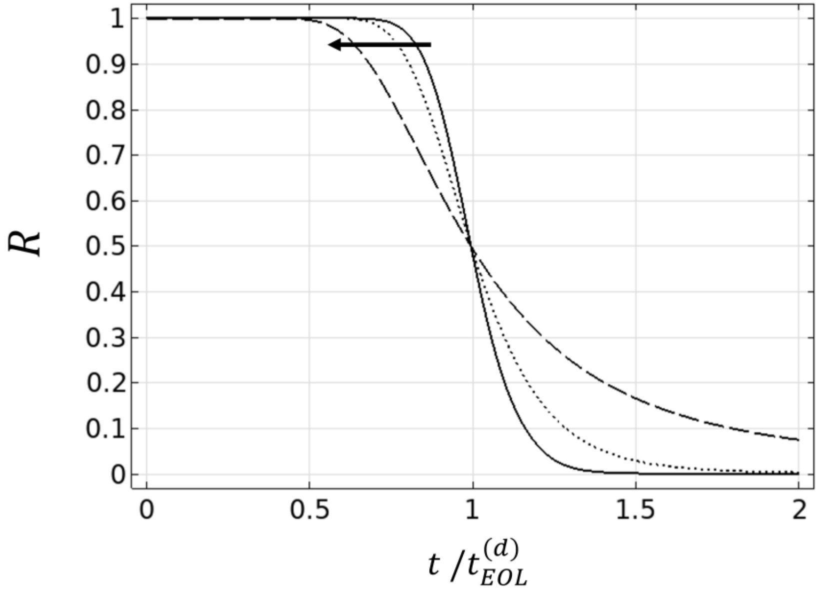
- 2 Langevin equations

$$\frac{d\mu_1}{dt} = \mu_2 + \sqrt{2D_1}\xi_1(t)$$

$$\frac{d\mu_2}{dt} = \sqrt{2D_2}\xi_2(t)$$

- Fokker-Planck (2d, in rectangle)

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial \mu_1}\left(D_1 \frac{\partial p}{\partial \mu_1} - \mu_2 p\right) + \frac{\partial}{\partial \mu_2}\left(D_2 \frac{\partial p}{\partial \mu_2}\right).$$



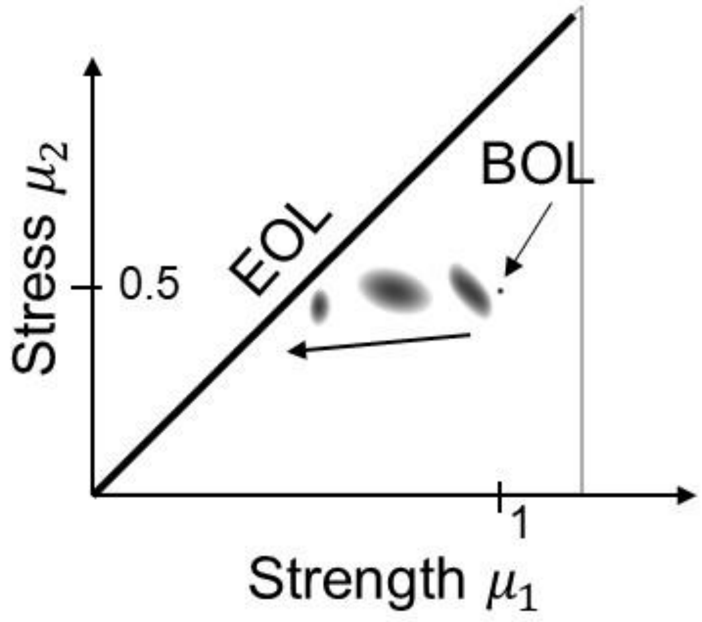
Early arrival by higher velocity tail leads to decrease of reliability before mean time to failure

Variable Failure Boundaries and Stress-Strength Reliability

What if μ_{EOL} is variable ? → Consider strength as additional stochastic aging model variable

- Example: stress-strength reliability
  - strength μ_1
  - stress μ_2

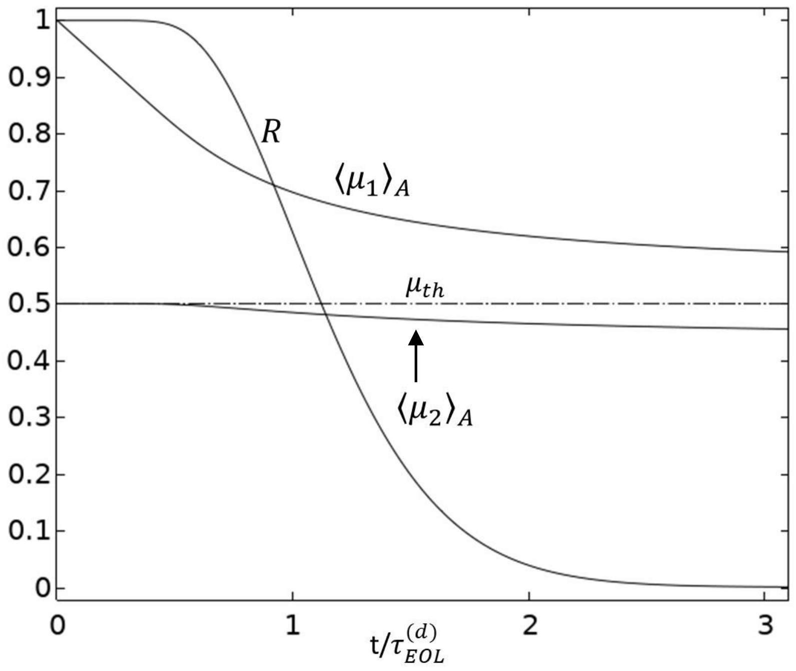
Stress dependent strength degradation



FEM Simulation (Comsol)

- Langevin equations

$$\frac{d\mu_1}{dt} = f_1(\mu_1,\mu_2) + \sqrt{2D_1}\xi_1(t)$$
$$\frac{d\mu_2}{dt} = f_2(\mu_1,\mu_2) + \sqrt{2D_2}\xi_2(t)$$



Example with a degradation acceleration threshold μ_{th}

- Fokker-Planck (Ω_A = triangle)

$$\frac{\partial p}{\partial t} = \sum_{k=1}^{2} \frac{\partial}{\partial \mu_k}\left(\frac{\partial(D_k p)}{\partial \mu_k} - f_k p\right).$$

Acceptance region averages

$$\langle y\rangle_A = \frac{1}{R}\int_{\Omega_A} y(\vec{\mu})p\,d\mu$$

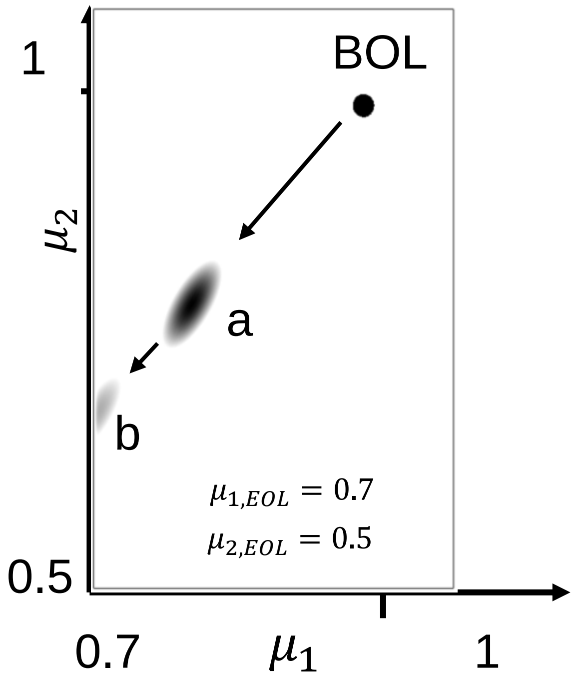
Including Chance Failures and Correlation

What if chance failures (statistical failures during useful life) need to be included in addition to wear-out (degradation) failures?
→ generalize FTE by adding sink term (→ decay rate, hazard)

❑ Example: Battery Aging

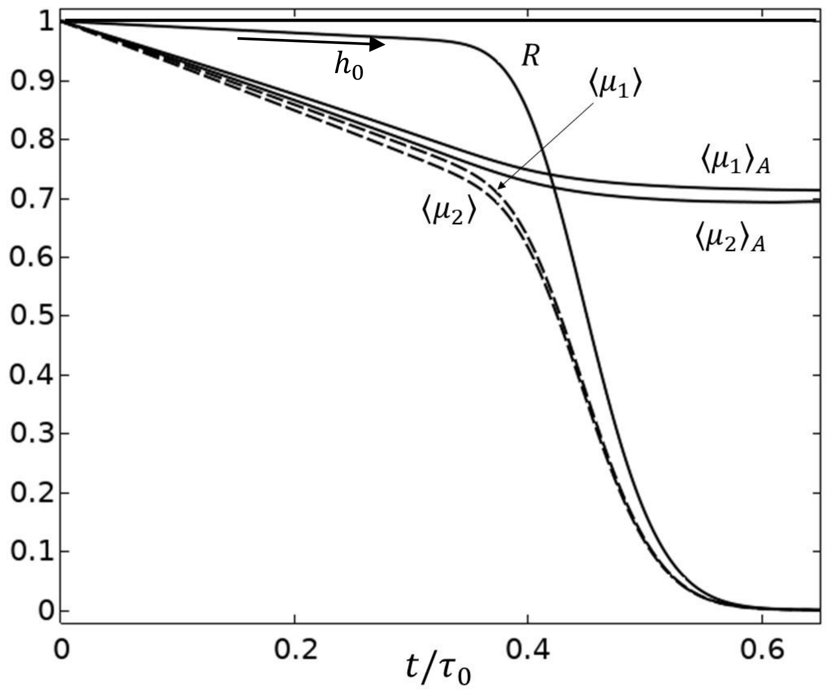
- Normalized capacity μ_1
- Normalized inverse ESR μ_2

FEM Simulation (Comsol)



❑ Langevin equations with nondiagonal *g*)

$$\frac{d\mu_1}{dt} = f_1(\mu_1,\mu_2) + \sqrt{2D_1}\xi_1(t)$$
$$\frac{d\mu_2}{dt} = f_2(\mu_1,\mu_2) + \sqrt{2D_2}\xi_2(t) + \gamma\sqrt{2D_1}\xi_1(t)$$



❑ Fokker-Planck with sink (in rectangle)

$$\frac{\partial p}{\partial t} = \sum_{i=1}^{2} \frac{\partial}{\partial \mu_i}\left(\sum_{j=1}^{2} \frac{\partial}{\partial \mu_j}(D_{ij}p) - f_i p\right) - h_0 p$$

Local conservation law as before, now with γ –dependent cross diffusion (non-diagonal diffusion matrix)

decay rate hazard h_0

Acceptance region averages

$$\langle y\rangle_A = \frac{1}{R}\int_{\Omega_A} y(\vec{\mu})p\, d\mu$$

Acceptance region integrals

$$\langle y\rangle = \int_{\Omega_A} y(\vec{\mu})p\, d\mu$$

Summary and Conclusion

- ❑ Convenient framework for prognostics: Fokker-Planck Equation (FPE)

- ❑ Prediction step
  - Degradation modeling with Langevin Equation (deterministic and stochastic part)
  - End of Life (EOL) boundaries in acceptance region of aging model variable space (or condition space)
  - FPE solved for probability distribution in acceptance region with absorbing EOL boundary conditions
  - Reliability calculated by integration of probability density in acceptance region
  - Determination of remaining useful life (RUL) from prescribed reliability limit value

- ❑ Correction step
  - Update probabilities with Bayesian inference when additional information is obtained

- ❑ Extensions
  - Additional auxiliary stochastic aging variables can be introduced (e.g., variable failure thresholds, colored noise, …) (colored noise: not discussed in this presentation)
  - Chance failures can be modeled with a sink term in the FPE

- ❑ For dimension ≤ 3 standard commercial PDE simulation tools (here Comsol, Finite Element Method) can be used for arbitrarily coupled and nonlinear aging models.

HITACHI Inspire the Next

References cited in article

I. Bazovsky, *Reliability Theory and Practice*, (orig. 1961) Dover Publications Inc., Mineola, New York, 2004.

K. C. Kapur and M. Pecht, *Reliability Engineering*, Series in Systems Engineering and Management, A. P. Sage ed., Wiley, New Jersey, 2014.

H. M. Elattar, H. K. Elminir, and A. M. Riad, “Prognostics: a literature review”, *Complex Intell. Syst.*, Vol. 2, 125-154, 2016.

J. Li, S. Peng, Y. Li, W. Jiang, “A review of condition-based maintenance: Its prognostic and operational aspects”, *Front. Eng. Manag.*, Vol 7, 323-334, 2020.

L. Liao and F. Köttig, “Review of Hybrid Prognostics Approaches for Remaining Useful Life Prediction of Engineered Systems, and an Application to Battery Life Prediction”, *IEEE Transactions on Reliability*, Vol. 63, No. 1, 191-207, 2014.

H. A. Bjaili, A. M. Rushdi, “Prognostics and Health Monitoring Methodologies and Approaches: A Review”, *Journal of Engineering Research and Reports*, Vol. 18, No. 4, 30-50, 2020.

M. Soleimani, F. Campean, D. Neagu, “Diagnostics and Prognostics for Complex Systems: A Review of Methods and Challenges”, *Quality and Reliability Engineering International.*, Vol. 37, No. 8, 3746-3778, 2021.

A.H. Rawicz and D. Girling, “Application of expert systems to systems reliability evaluation”, *Microelectron. Reliabi.*, Vol. 35 (9) 1309-1320 (1995).

O. Fink, Q. Wang, M. Svensén, P. Dersin, W.-J. Lee, M. Ducoffe, “Potential, challenges and future directions for deep learning in prognostics and health management applications”, *Engineering Applications of Artificial Intelligence*, Vol. 92, 103678, 2020.

W. Vermeer, G. Mouli, and P. Bauer, “A Comprehensive Review on the Characteristics and Modeling of Lithium-Ion Battery Aging”, *IEEE Transactions on Transportation Electrification*, Vol. 8, No. 2, 2205-2232, 2022.

X. Hu, L. Xu, X. Lin, M. Pecht, “Battery Lifetime Prognostics”, *Joule*, Vol. 4, 310-346, 2020.

S. Wang, S. Jin, D. Deng, C. Fernandez., “A Critical Review of Online Battery Remaining Useful Lifetime Prediction Methods”, *Front. Mech. Eng.*, Vol. 7, No. 1, 719718, 2021.

S. Zhang, Q. Zhai, X. Shi, and X. Liu “A Wiener Process Model With Dynamic Covariate for Degradation Modeling and Remaining Useful Life Prediction”, *IEEE Transactions on Reliability*, Vol. 72, No. 1, 214-223, 2023.

Z. Li., H. Li, F. Lin, Y. Chen, D. Liu, B. Wang, Q. Zhang, and W. He, “Lifetime Prediction of Metallized Film Capacitors Based on Capacitance Loss”, *IEEE Transactions on Plasma Science*, Vol. 41, No. 5, 1313-1318, 2013.

R. Gallay, “Metallized Film Capacitor Lifetime Evaluation and Failure Mode Analysis”, *Proceedings of the CAS-CERN Accelerator School: Power Converters*, Baden, Switzerland, CERN-2015-003, 45-56, 2015.

X.-S. Si, W. Wang, C.-H. Hu, D.-H. Zhou, M. G. Pecht, “Remaining Useful Life Estimation Based on a Nonlinear Diffusion Degradation Process”, *IEEE Transactions on Reliability*, Vol. 61, No. 1, 50-67, 2012.

M. Fan, Z. Zeng, E. Zio, R. Kang, and Y. Chen “A Sequential Bayesian Approach for Remaining Useful Life Prediction of Dependent Competing Failure Processes”, *IEEE Transactions on Reliability*, Vol. 68, No. 1, 317-329 (2019).

C. Park and W. Padgett, “Accelerated Degradation Models for Failure Based on Geometric Brownian Motion and Gamma Processes”, *Lifetime Data Analysis*, Vol. 11, 511–527, 2005.

N. G. van Kampen, *Stochastic Processes in Physics and Chemistry*, North-Holland Publishing Comp. Amsterdam, 1981.

M. San Miguel and R. Toral, “Stochastic Effects in Physical Systems”, *Instabilities and Nonequilibrium Structures VI*, E. Tirapegui, J. Martinez, R. Tiemann (eds), Nonlinear Phenomena and Complex Systems, Vol. 5. Springer, Dordrecht, Netherlands 35-127, 2000.

Y. Deng, “Degradation Modeling Based on a Time-dependent Ornstein-Uhlenbeck Process and Prognosis of System Failures”, *PhD Thesis*, Troyes University of Technology, 2015.

Y. Deng, A. Barros, and A. Grall, “Degradation Modeling Based on a Time-Dependent Ornstein-Uhlenbeck Process and Residual Useful Lifetime Estimation”, *IEEE Transactions on Reliability*, Vol. 65, No. 1, 126-140, 2016.

D. Wang and K-L. Tsui, “Brownian motion with adaptive drift for remaining useful life prediction: Revisited”, *Mechanical Systems and Signal Processing*, 99, 691–701, 2018.

P. Sura, M. Newman, C. Penland, P. Sardeshmuk, “Multiplicative Noise and Non-Gaussianity: A Paradigm for Atmospheric Regimes?”, *Journal of the Atmospheric Sciences*, Vol. 62, 1391-1409, 2005.

M. Prasad, G. Reddy, A. Srividya, and A. Verma, “Stochastic Reliability Analysis Using Fokker Planck Equations”, *Fourth national conference on nuclear reactor technology: emerging trends in nuclear safety*, NRT4-2011 Organized by BARC & BRNS (2009); only abstract exists.

Y. Lei, N. Li, S. Gontarz, J. Lin, S. Radkowski, and J. Dybala, “A Model-Based Method for Remaining Useful Life Prediction of Machinery”, *IEEE Transactions on Reliability*, Vol. 65 , No. 3, 1314-1326, 2016.

J. Náprstek and R. Král, “Multi-dimensional Fokker-Planck equation analysis using the modified finite element method”, *Journal of Physics: Conference Series* 744, 012177, 2016.

P. S. Maybeck, “Stochastic Models, Estimation, and Control”, Vol. I, *Academic Press Inc. N.Y.*, 1979.

B. Tiger and K. Weir, “Stress-Strength Theory and Its Transformation into Reliability Functions”, *Second Annual Symposium on the Physics of Failure in Electronics*, Chicago, IL, USA, 94-101, 1963.

J. Wilhelm, S. Seidlmayer, P. Keil, J. Schuster, A. Kriele, R. Gilles, A. Jossen, “Cycling capacity recovery effect: A coulombic efficiency and postmortem study”, *Journal of Power Sources*, 365, 327, 2017.

Z. Wang, Y. Chen, Z. Cai, Z. Chang, W. Tao, “Remaining Useful Lifetime Prediction for the Equipment with the Random Failure Threshold”, *IEEE 2019 Prognostics & System Health Management Conference—Qingdao*, 2019.

F. Rudsari, A. Razi-Kazemi, M. Shoorehdeli, “Fault Analysis of High-Voltage Circuit Breakers Based on Coil Current and Contact Travel Waveforms Through Modified SVM Classifier”, *IEEE Transactions on Power Delivery*, Vol. 34, No. 4, 2019.

T. Paez, “Introduction to Model Validation”, No. SAND2008-7314C. Sandia National Lab. (SNL-NM), Albuquerque, NM (United States), 2008.

Hitachi Energy

Hitachi Energy

HITACHI
Inspire the Next