A Novel Physics-informed Neural Networks Approach for Efficient Multimodal Mapping and Inversion of Vibrations

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Overview

- Introduction and Motivation
- PINNs for Modelling and Inversion of Vibrations
- Proposed PINN Methodology
- Results
- Concluding Remarks

Flanders Make
Research Institute to support the Digitization of Industry

Clustered around 3 competences

Manufacturing industry

Industry with Manufacturing Challenges

End-to-end design operation

Leverage data and knowledge throughout the product lifecycle and value chain

Motion products

Validated products & product architectures

Production

Validated assembly solutions, using robotics & automation

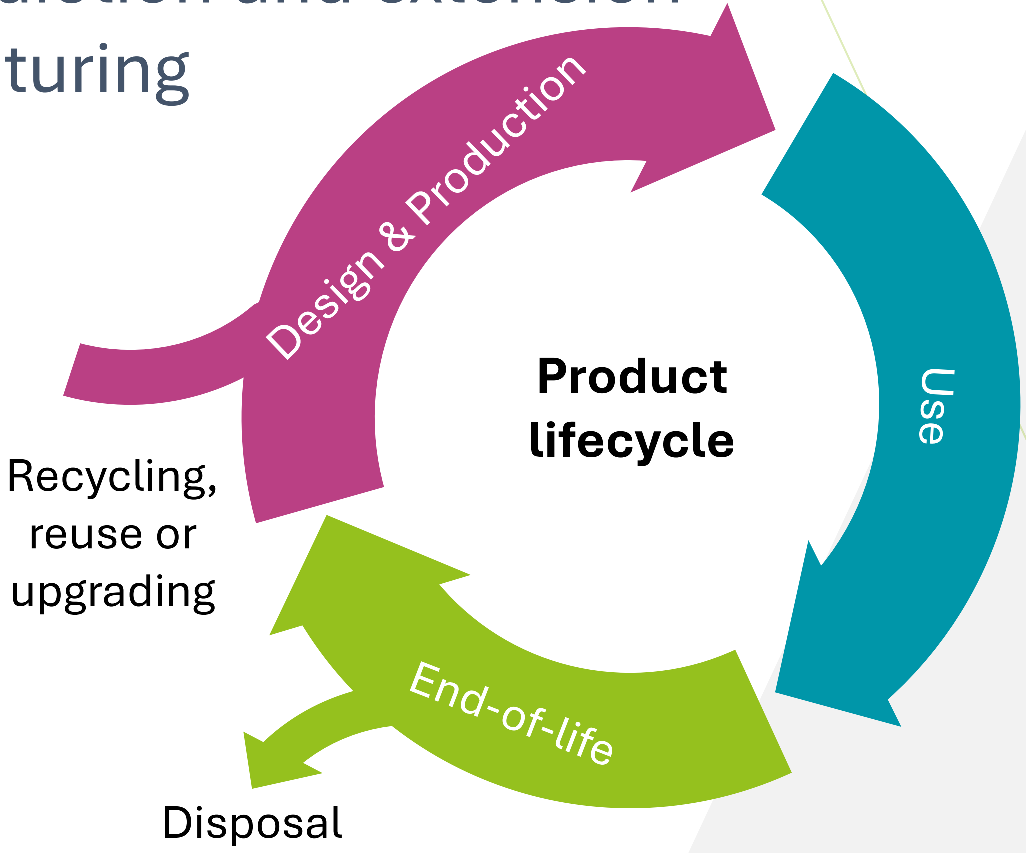
Tech. Domain: Sensing and Monitoring

- Manufacturing process monitoring and correction
- Condition monitoring, lifetime prediction and extension
- End-of-line quality and remanufacturing

Physics-aware ML

Diagnosis, Prognosis and Control

- ➢ Sparse sensor locations
- ➢ Inaccessible locations
- ➢ Weakly known physics
- ➢ ….

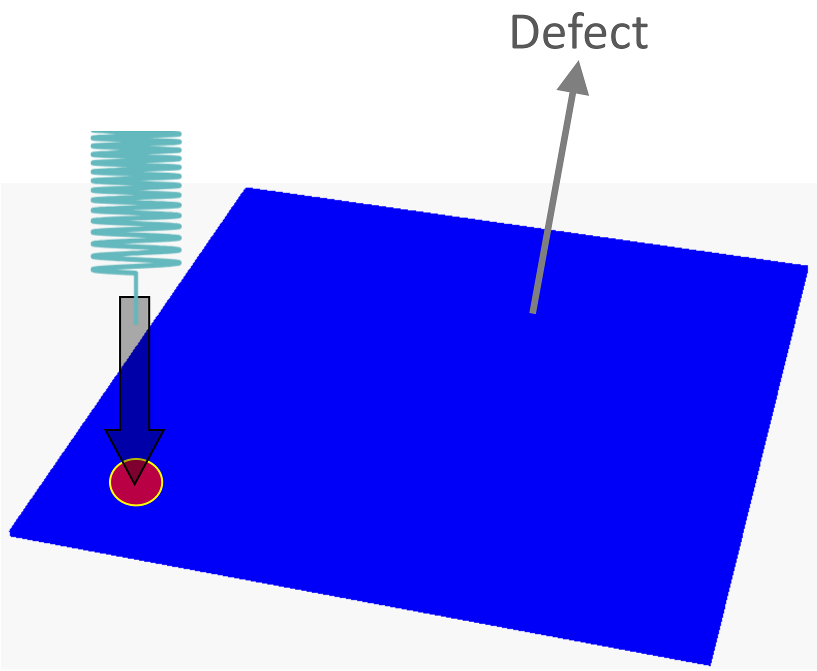


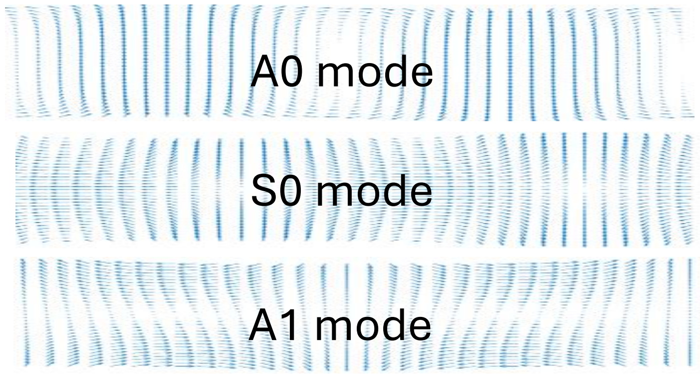
Characterization with Vibrations and Elastic Waves

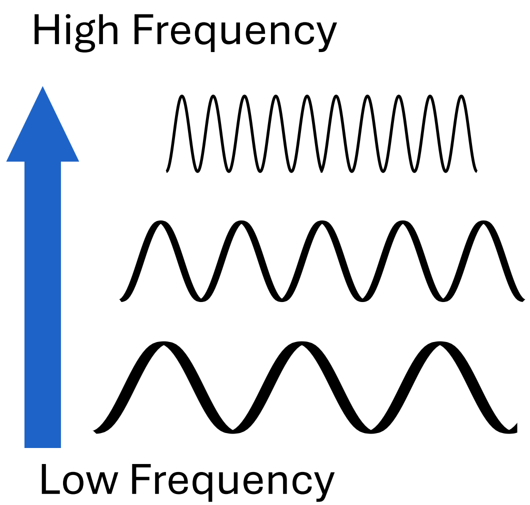
Inject Vibrations

Multimodal and Multiscale Interaction

Global and Local Characterization







Material, thickness, …

- Modal frequencies
- Mode shapes
- Propagation speed
- Dispersion
- …

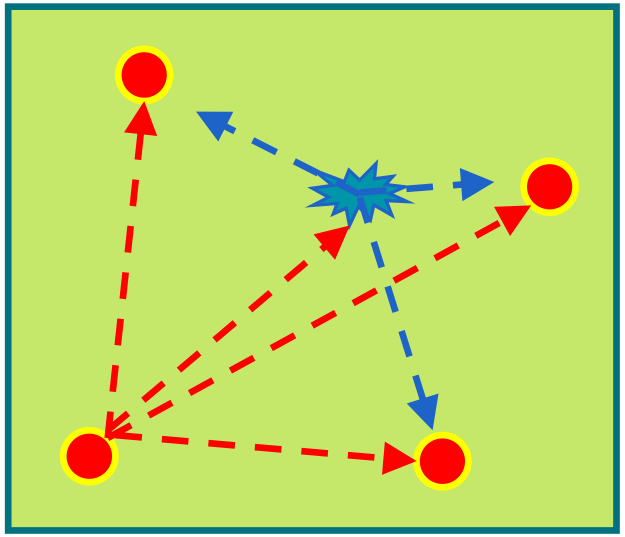
Defect

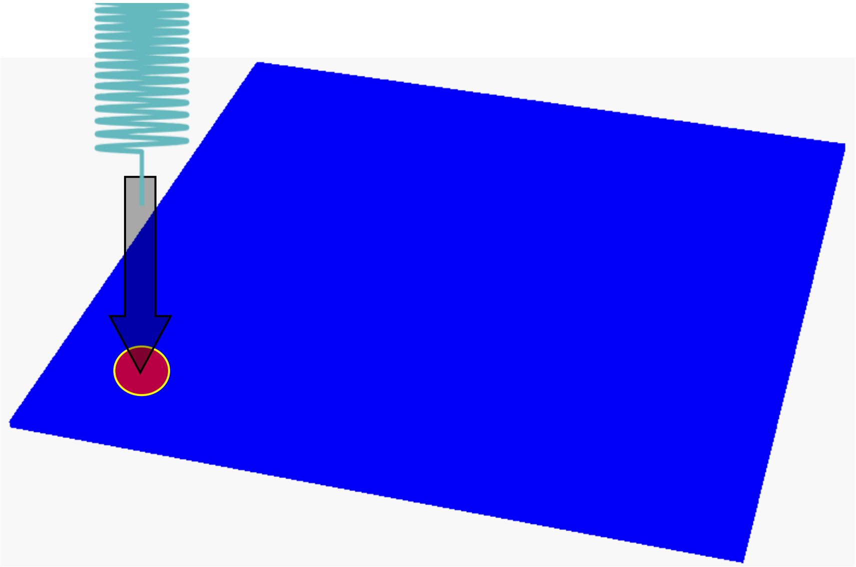
Localized abnormal response

- Density
- Stiffness
- Thickness
- Nonlinearity
- …

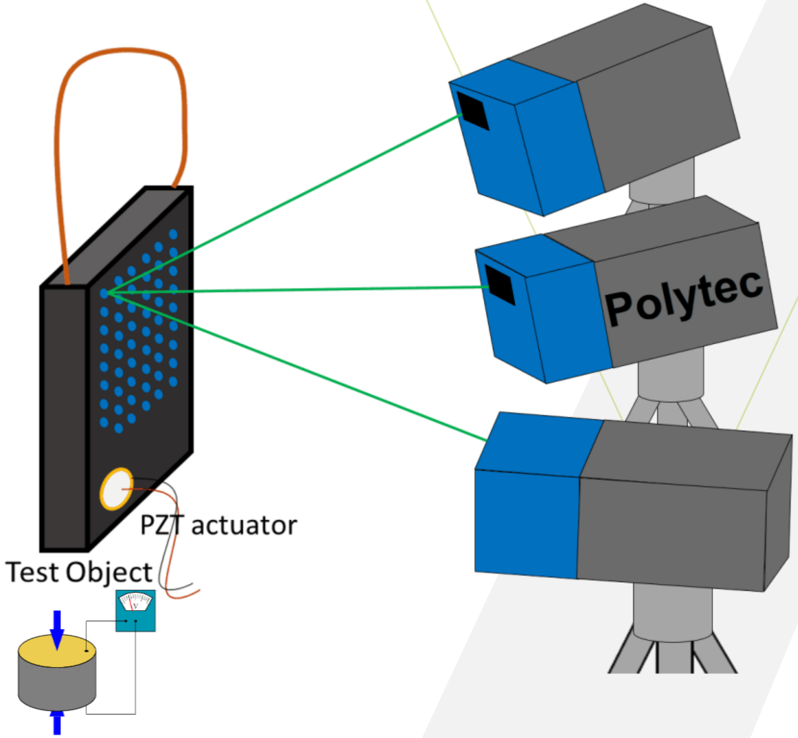
Sparse vs. Full-field Data

Sparse Sensor Network

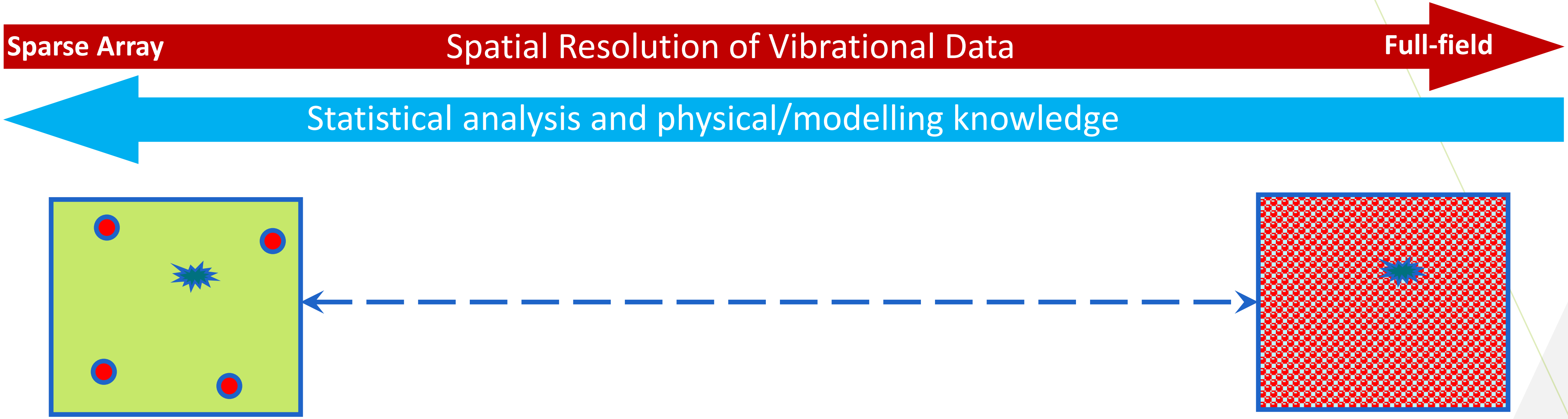




Full-field (SLDV)

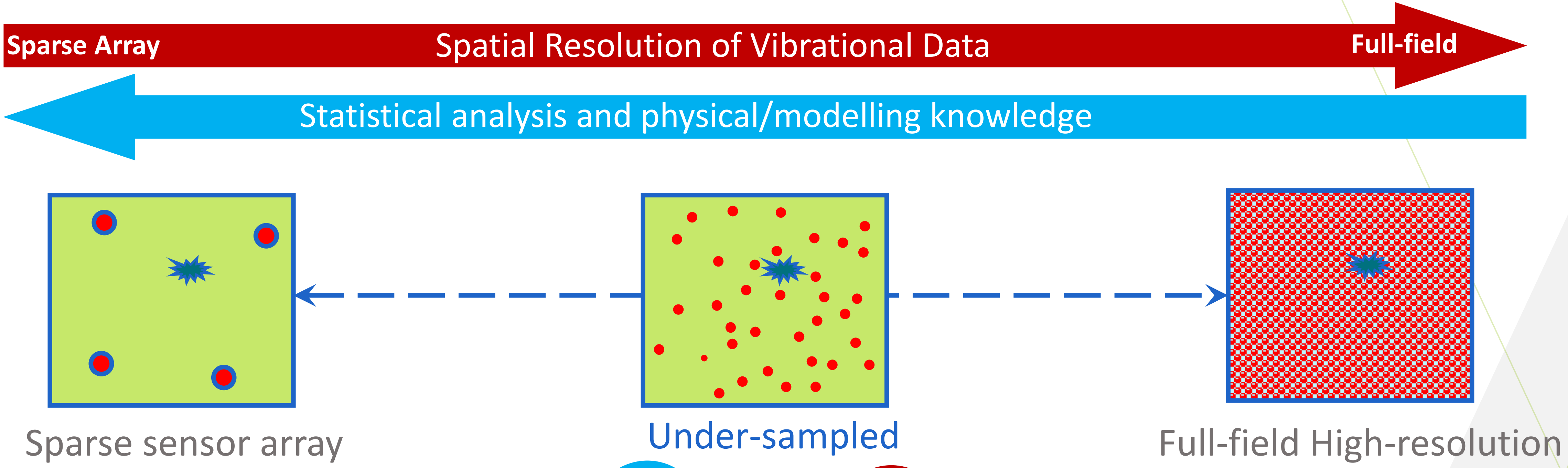


Sparse/Full-field Data

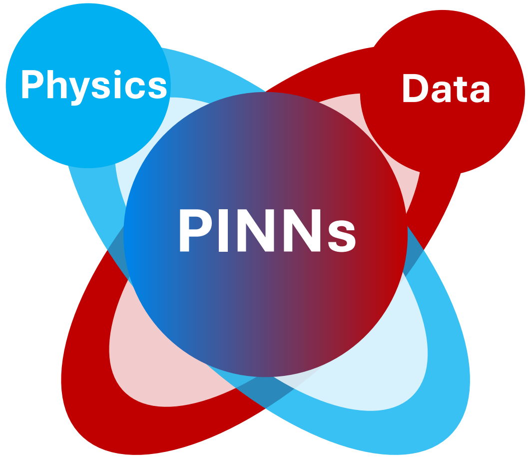


Sparse sensor array	Full-field High-resolution
✓ Low cost	✓ Abundant data
✓ Fast	✓ Localization, Quantification
➢ Physics knowledge	× Tedious scanning
➢ Probabilistic	× Slow for online application
	× Inaccessible areas

Under Sampled→ Physics-informed Neural Networks (PINNs)

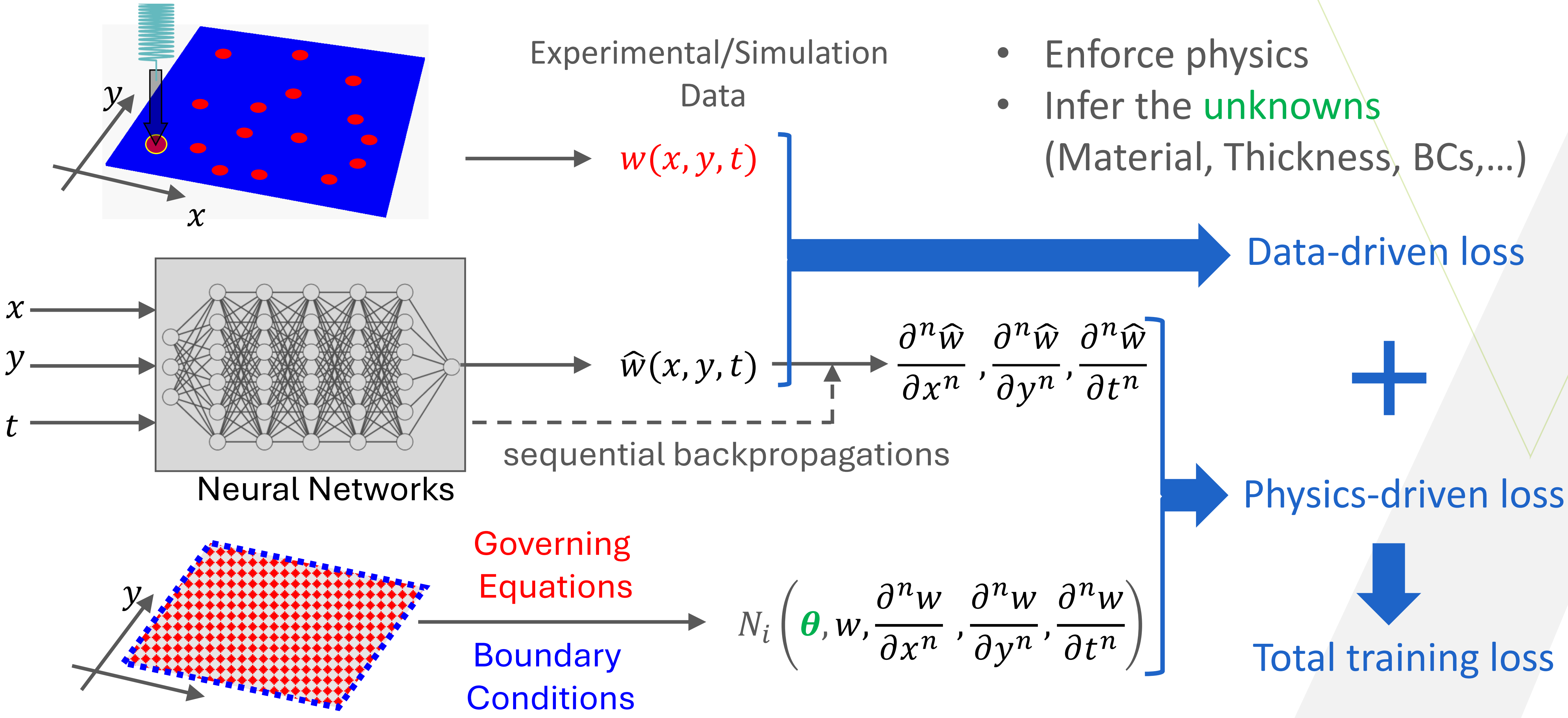


- Model-based Inversion
- Compressed Sensing
- Blackbox Regression (NNs)
- …



- High expressivity of NNs
- Seamless fusion of data and physics
- Enforce/Discover Physics

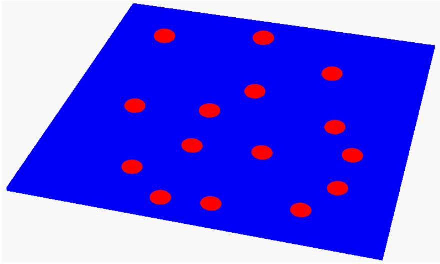
Physics-informed Neural Networks (PINNs)



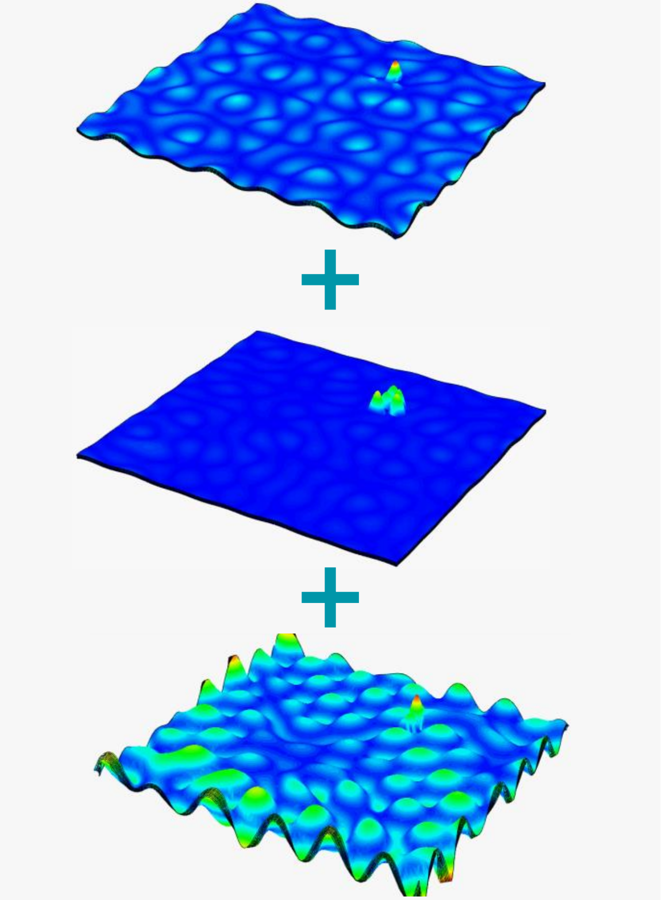
Objectives and Scope

1. Full-field Reconstruction of Modal Responses
2. Identify Elastic Coefficients

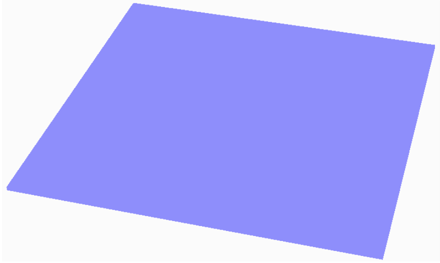
Broadband Excitation



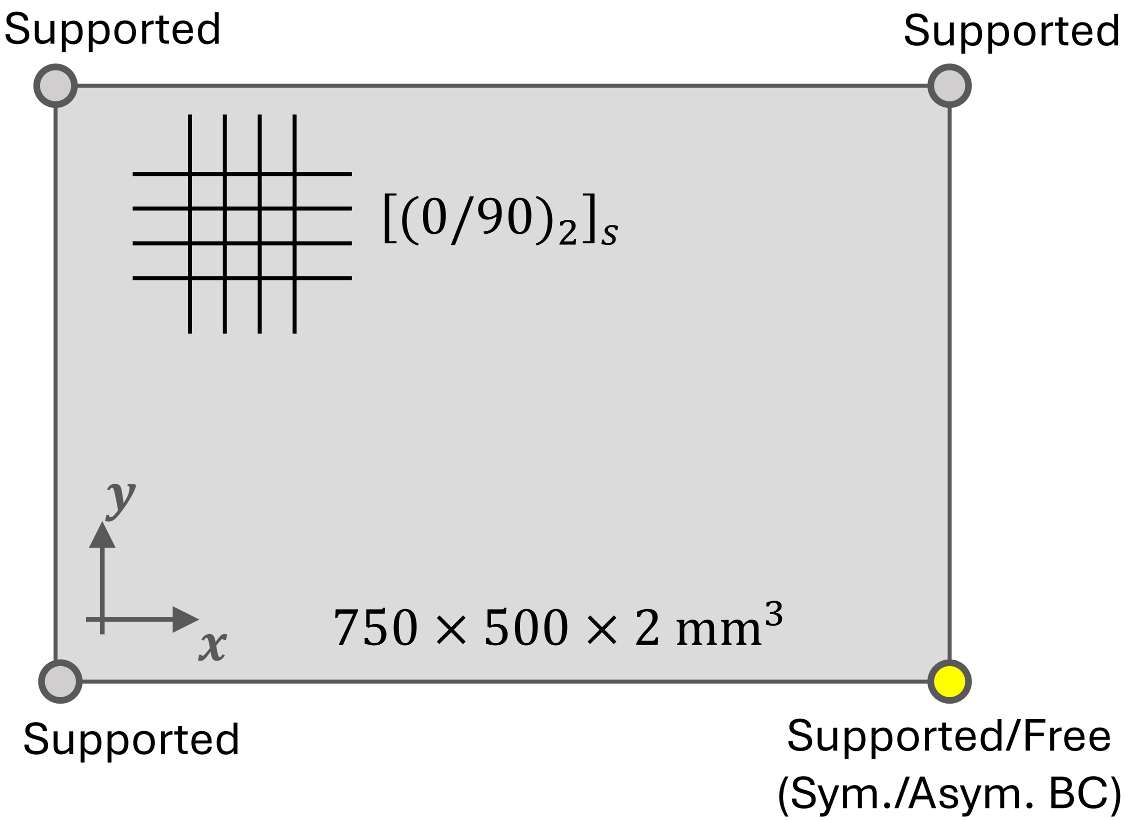
- Sparse data
- Modal Frequency Identification



- Localize defect ...
- Superposition
  (Linear time-invariant behavior)



Case Study: Bending Vibrations of a Cross-ply Composite Laminate



Classical Laminate Theory

Single DOF: Bending deflection w

Orthotropic bending stiffness: D_{ij}

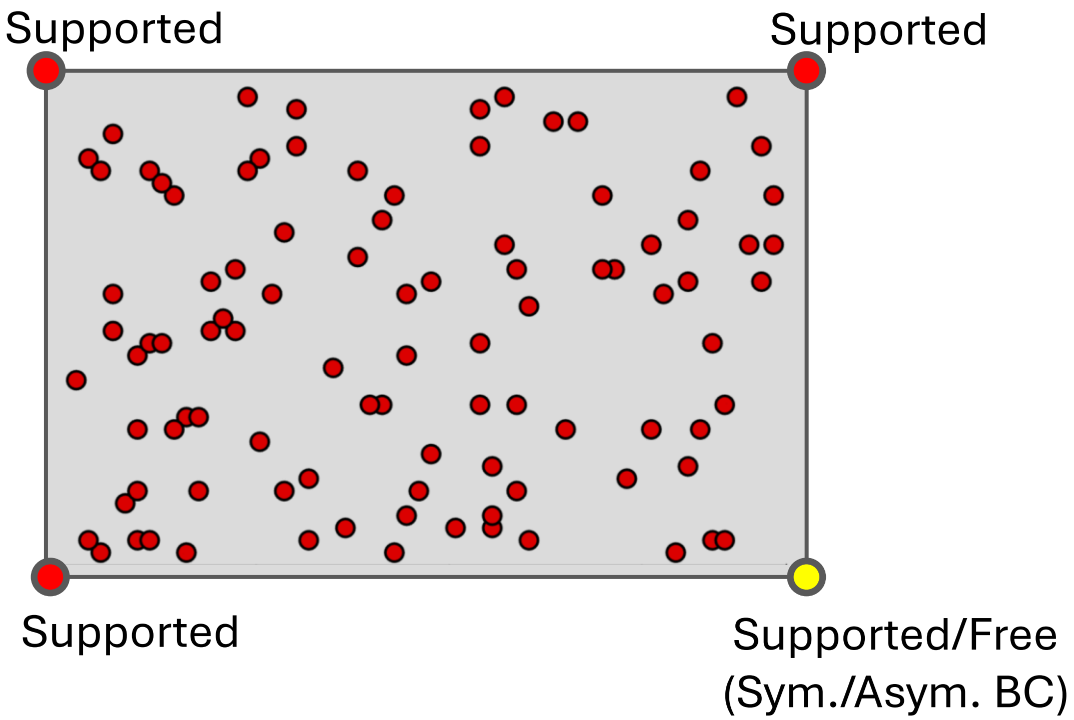
Bending inertia: I_d, I_r

$N = 4^{\text{th}}$ order PDE

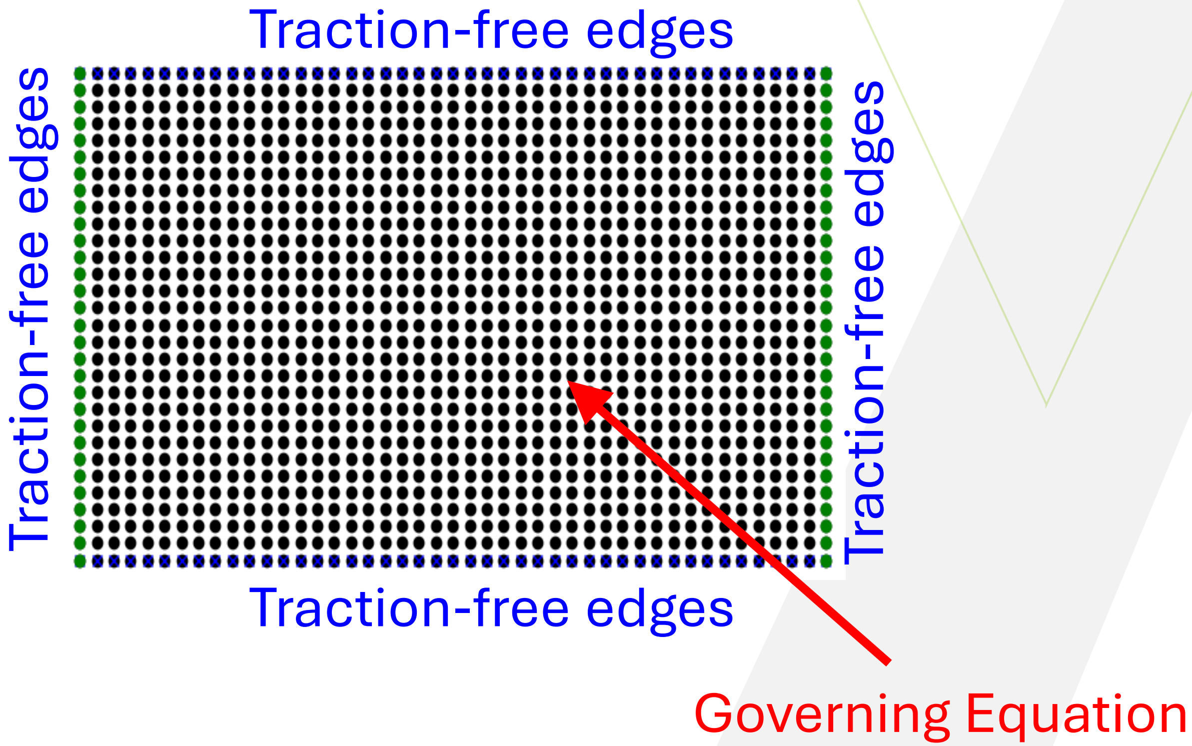
$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} - I_d\omega^2 w + I_r\omega^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0$$

Case Study: Bending Vibrations of a Cross-ply Composite Laminate

Virtual test data
(FE Modal Analysis
+ White Gaussian Noise SNR=20 dB)



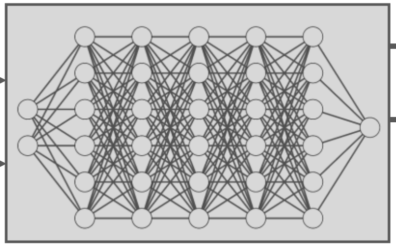
Physics
(on uniform/random collocation points)



PINN for A Single Mode Shape

N sequential backpropagations

x, y →



Neural Network

→ $w(x,y)$ → $\frac{\partial^n w}{\partial x^n}, \frac{\partial^n w}{\partial y^n}$

$n \in \{1,2,3,\dots,N\}$

Mode shape: $\frac{\partial^n w}{\partial t^n} = 0$

$\mathcal{L}_{Total}$ = $\mathcal{L}_{Data}$ + $\mathcal{L}_{Physics}$

(Total loss) (Data-driven loss) (Physics-informed losses)

An average stiffness scaling factor:

$D = \bar{\boldsymbol{\theta}}_{\boldsymbol{D}}\{D_{11}^0 \quad D_{22}^0 \quad D_{12}^0 \quad D_{66}^0\}$

$\bar{\theta}_D$:Initial Guess 0.5 (➔True 1.0)

Challenges: Imbalance of Elastic and Inertial Terms

- High-order elastic terms versus low-order inertial terms ➔ Trivial solution by near-zero elastic coefficients!

Elastic Terms — Inertial Terms

$$\left\{D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4}\right\} + \left\{-I_d\omega^2 w + I_r\omega^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right\} = 0$$

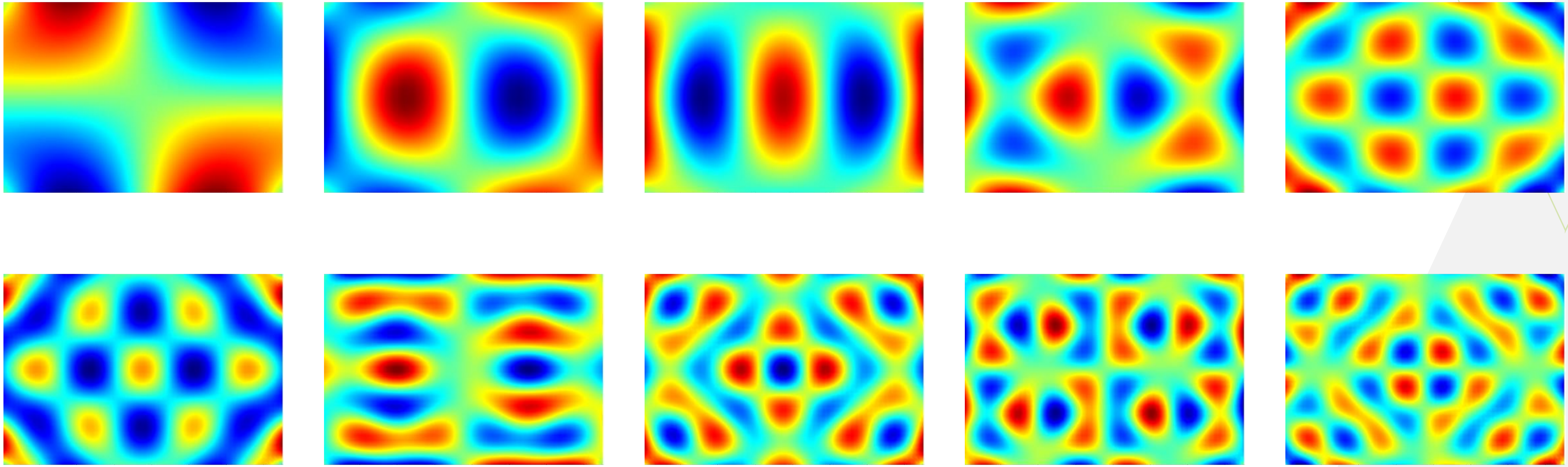
- ➢ Order reduction by reparameterization: Auxiliary output variables, and loss terms

$$D_{11}\frac{\partial^2 w_{xx}}{\partial x^2} + 2(D_{12} + 2D_{66})\frac{\partial^2 w_{xx}}{\partial y^2} + D_{22}\frac{\partial^2 w_{yy}}{\partial y^2} - I_d\omega^2 w + I_r\omega^2(w_{xx} + w_{yy}) = 0$$

$$Y = [w, w_{xx} = \frac{\partial^2 w}{\partial x}, w_{yy} = \frac{\partial^2 w}{\partial y^2}]$$

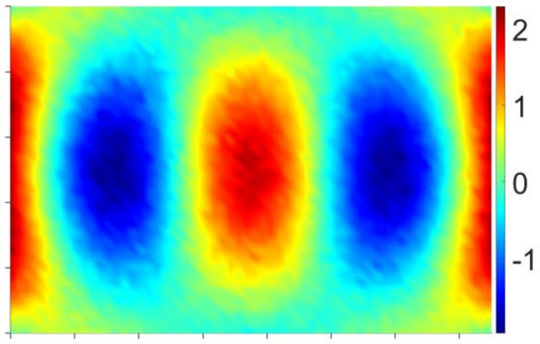
Challenges: Multi-scale Multi-modal Patterns

- Spectral Bias of NNs: Less complex (low frequency) modes
  - ➢Sine activation functions?
  - ➢Locally adaptive scale?



PINN for A Single Mode Shape

Mode Shape (150.01 Hz)

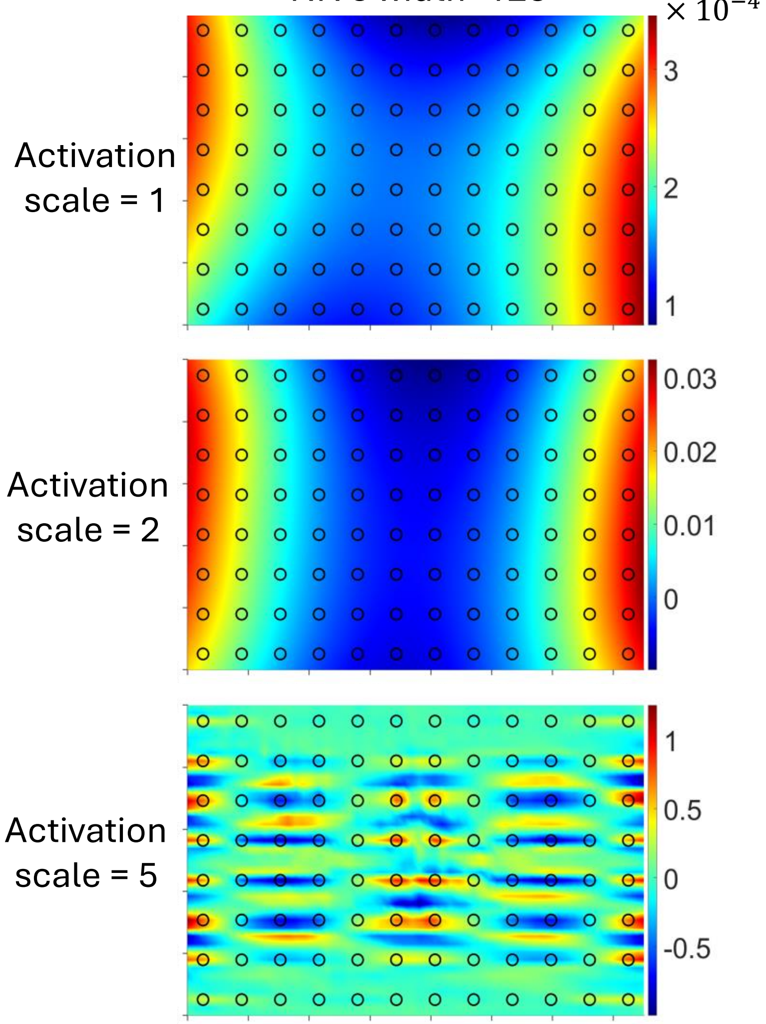


- Locally adaptive activation

  Scale: 1,2 and 5
- Hidden Depth = 5
- Width: 32, 128 and 512
- 12x8=96 Data points

Reconstruction

NN's width=128



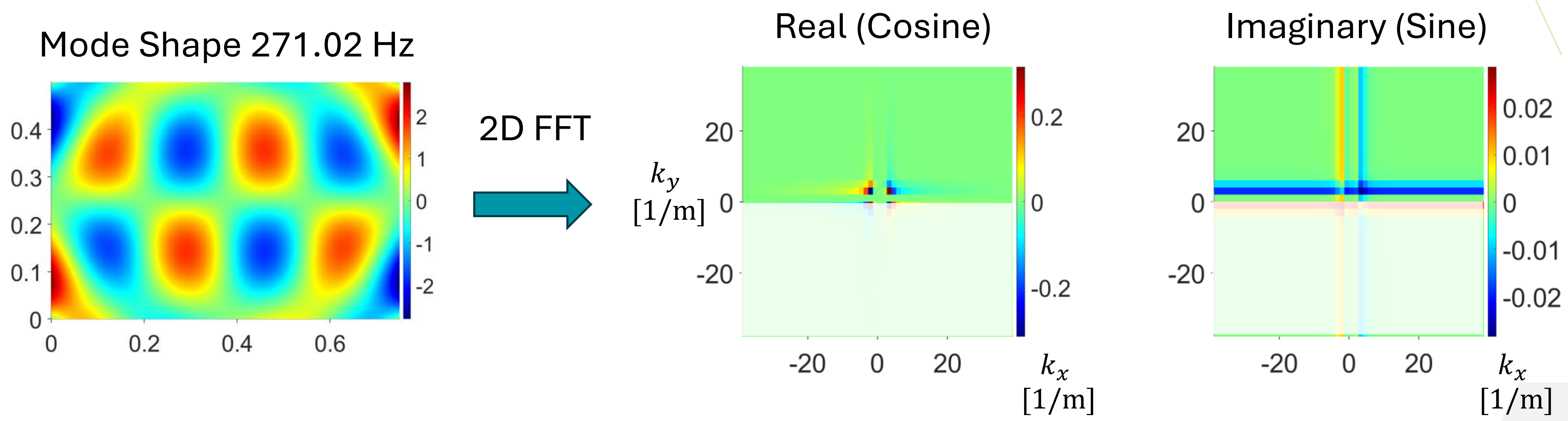
Identified Stiffness Scaling $\bar{\theta}_D$

NNs' Type	NN's Width	Activation Scale	$\bar{\theta}_D$
PINN	32	1	0.426
		2	0.375
		5	0.568
	128	1	0.437
		2	0.385
		5	0.451
	512	1	0.451
		2	0.387
		5	0.441

$\bar{\theta}_D$:Initial Guess 0.5 (➔True 1.0)

Spatial-domain ➔ Wavenumber-domain (k-Space)

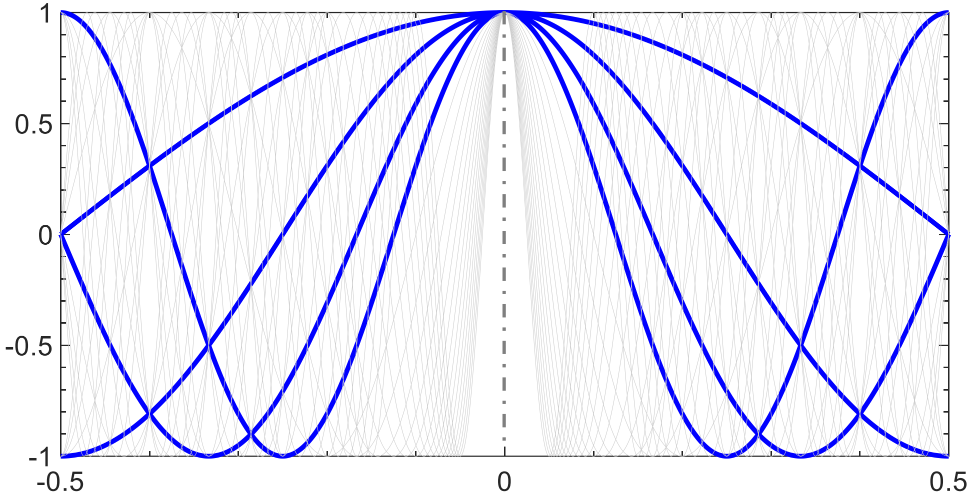
- Sparse and diagonally complex conjugate
- Multiscale spectral definition



$$\widetilde{w}(k_x, k_y, f) = \frac{1}{N_x \times N_y} \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} w(x(n_x), y(n_y), f) \exp\left(-2\pi i \left(k_x x(n_x) + k_y y(n_y)\right)\right)$$

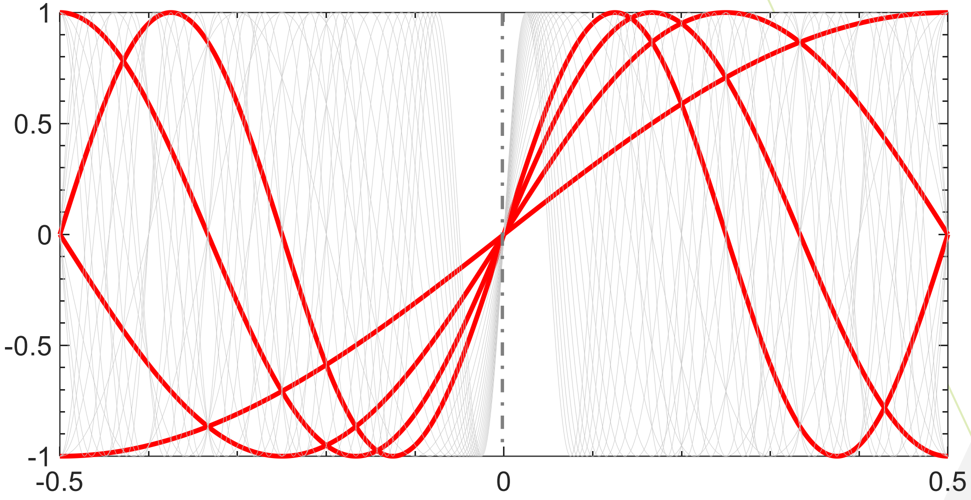
Reconstruction in k-space

Real (Symmetric)



Centralized and Normalized Spatial domain

Imaginary (Antisymmetric)



$$\tilde{w}(k_x, k_y, f) = \tilde{w}_{Re}(k_x, k_y, f) + i\, \tilde{w}_{Im}(k_x, k_y, f)$$

$$\begin{Bmatrix} \hat{w} \\ \dfrac{\partial^n \hat{w}}{\partial x^n} \\ \dfrac{\partial^n \hat{w}}{\partial y^n} \end{Bmatrix} = \sum_{m_x=-N_f\times B_x}^{N_f\times B_x} \sum_{m_y=-N_f\times B_y}^{N_f\times B_y} \begin{Bmatrix} 1 \\ (2\pi i k_x)^n \\ \left(2\pi i k_y\right)^n \end{Bmatrix} \tilde{w}\left(k_x(m_x), k_y(m_y), f\right) \exp\left(2\pi i\left(k_x(m_x)x + k_y(m_y)y\right)\right)$$

k-space PINN (k-PINN)

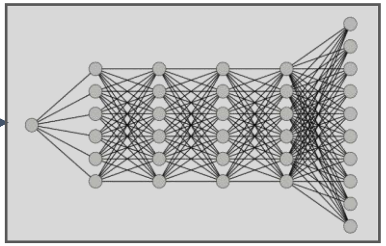
- Decode the vector of modal frequencies to their k-space solution
- Reconstruct mode shapes by spectral superposition (Inverse Fourier Transform)
- Add sparsity promoting loss term

✓ Multi-modal Mapping

✓ Tackle spectral bias

✓ No additional backpropagations

Modal freq. vector $\boldsymbol{f}$ →



Neural Network

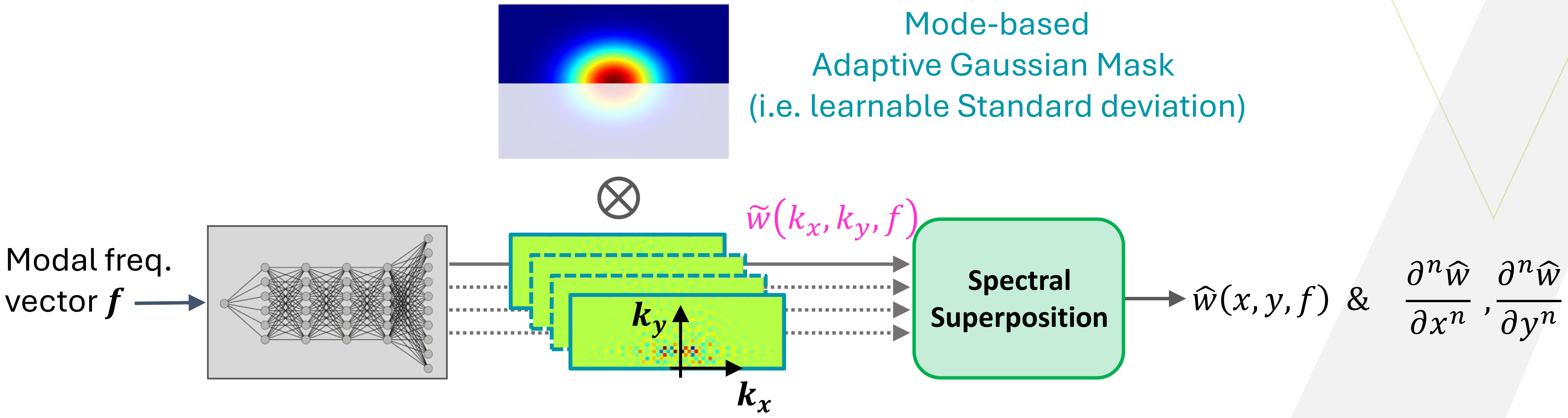
$\tilde{w}(k_x, k_y, f)$ → **Spectral Superposition** → $\hat{w}(x, y, f)$ & $\dfrac{\partial^n \hat{w}}{\partial x^n}, \dfrac{\partial^n \hat{w}}{\partial y^n}$

$$\mathcal{L}_{Total} = \mathcal{L}_{Sparse} + \mathcal{L}_{Data} + \mathcal{L}_{Physics}$$

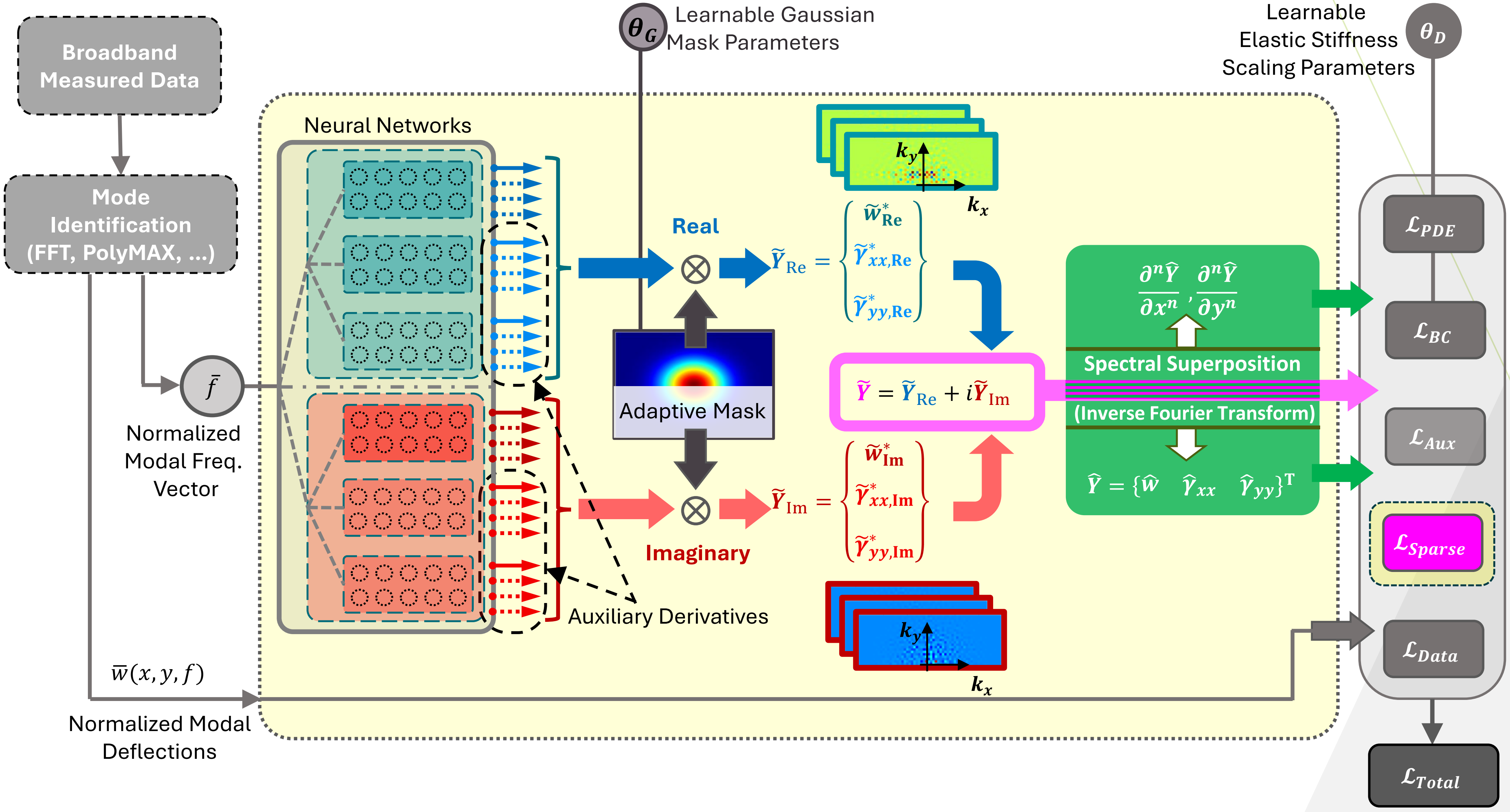
(Total loss) (Spectral sparsity loss) (Data-driven loss) (Physics-informed losses)

Adaptive Regularization of k-space

- Allocate a wideband k-space ➔ Redundant spectral components!
  - ➢ Apply a Gaussian prior (low-pass filter) to adaptively adjust the effective bandwidth for each mode

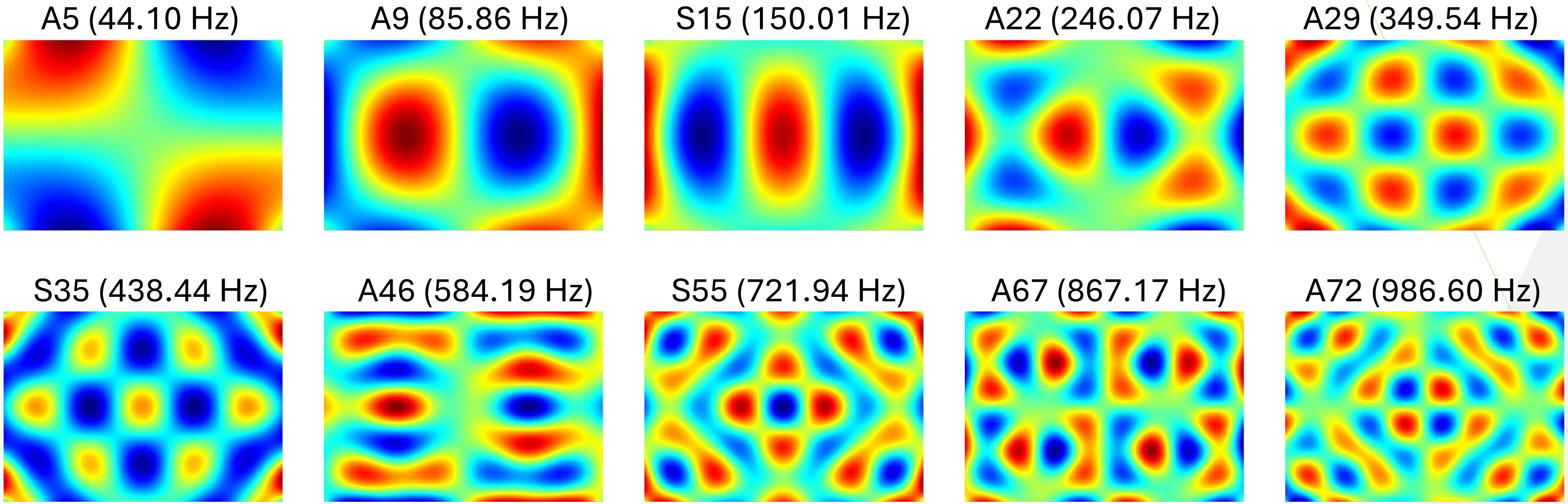


k-PINN



Broadband Measured Data
Mode Identification (FFT, PolyMAX, ...)
$\bar{f}$
Normalized Modal Freq. Vector
$\bar{w}(x, y, f)$
Normalized Modal Deflections
Neural Networks
θ_G
Learnable Gaussian Mask Parameters
Real
Imaginary
Adaptive Mask
Auxiliary Derivatives
$\tilde{Y}_{\mathrm{Re}} = \{\tilde{w}^*_{\mathrm{Re}}, \tilde{Y}^*_{xx,\mathrm{Re}}, \tilde{Y}^*_{yy,\mathrm{Re}}\}$
$\tilde{Y}_{\mathrm{Im}} = \{\tilde{w}^*_{\mathrm{Im}}, \tilde{Y}^*_{xx,\mathrm{Im}}, \tilde{Y}^*_{yy,\mathrm{Im}}\}$
k_y
k_x
$\tilde{Y} = \tilde{Y}_{\mathrm{Re}} + i\tilde{Y}_{\mathrm{Im}}$
$\frac{\partial^n \hat{Y}}{\partial x^n}, \frac{\partial^n \hat{Y}}{\partial y^n}$
Spectral Superposition
(Inverse Fourier Transform)
$\hat{Y} = \{\hat{w} \quad \hat{Y}_{xx} \quad \hat{Y}_{yy}\}^{\mathrm{T}}$
Learnable Elastic Stiffness Scaling Parameters
θ_D
$\mathcal{L}_{PDE}$
$\mathcal{L}_{BC}$
$\mathcal{L}_{Aux}$
$\mathcal{L}_{Sparse}$
$\mathcal{L}_{Data}$
$\mathcal{L}_{Total}$

Evaluate k-PINN: 10 Mode Shapes (Symmetric BCs)

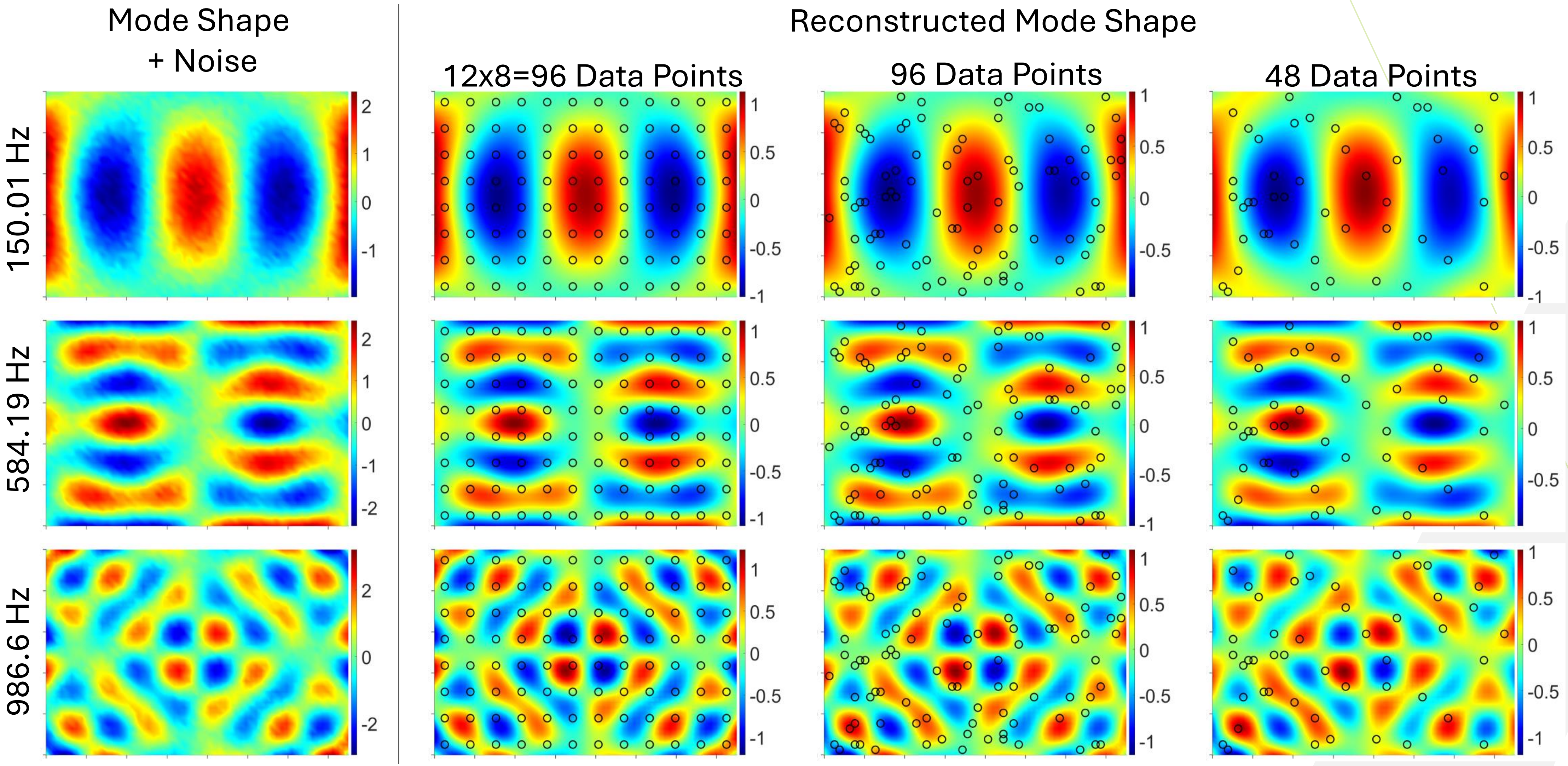


S: Symmetric Mode Shape

A: Anti-symmetric Mode Shape

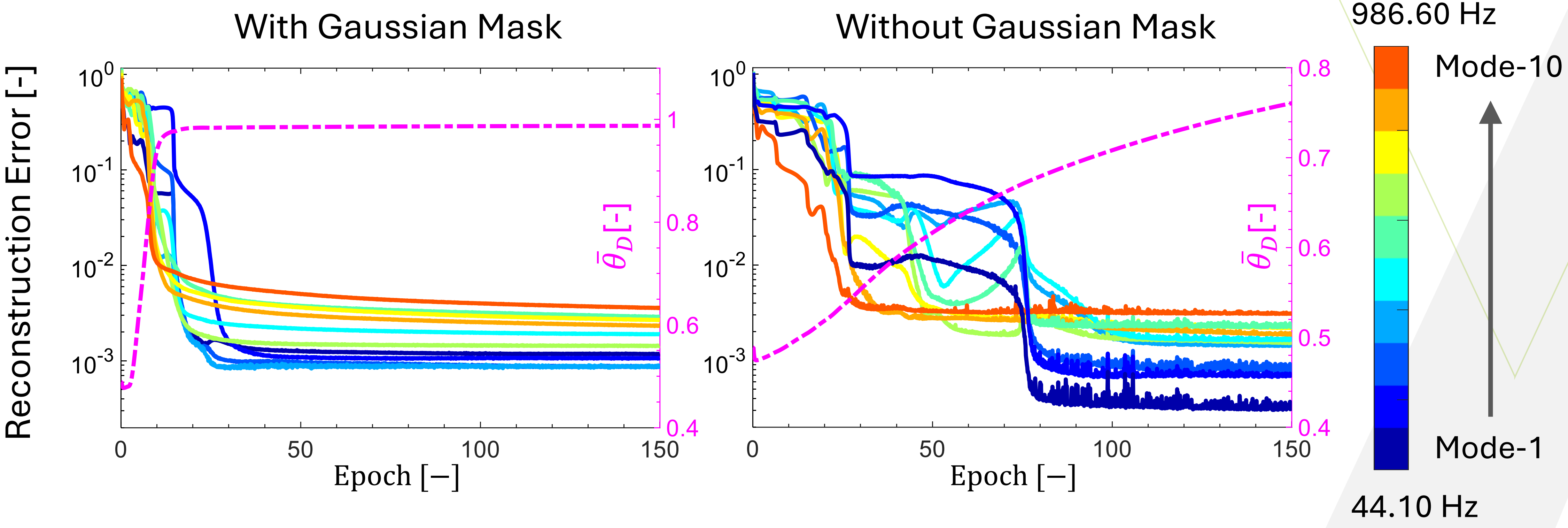
All four corners supported (Symmetric BC)

Full-field Reconstruction (Symmetric BCs)



Evaluate k-PINN: Physics-informed Training

- Dense Dataset: a uniform grid of 60x40=2400 Points



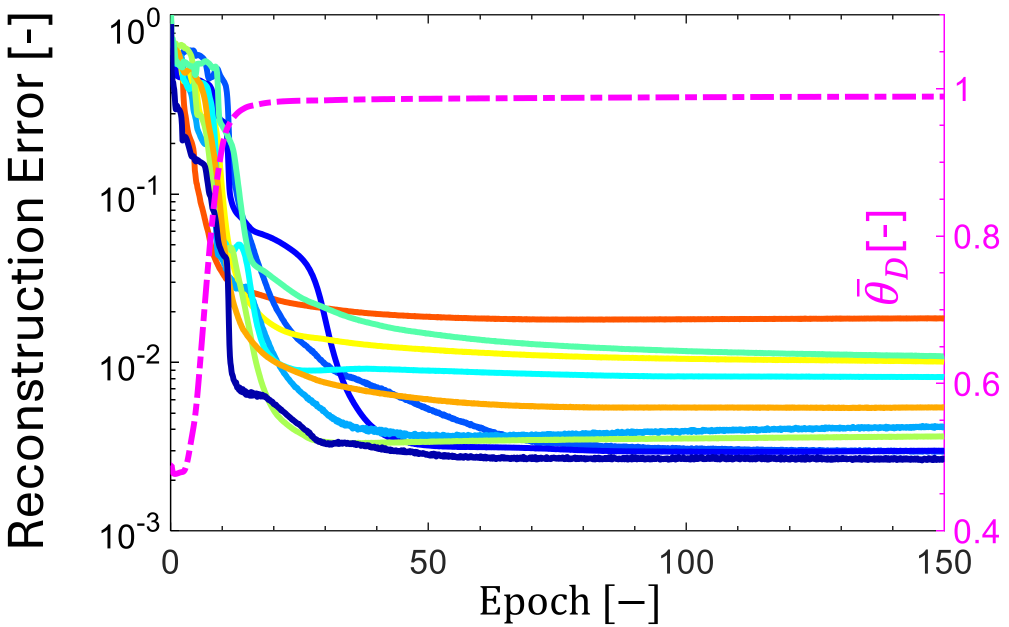
- Faster convergence by Gaussian Mask
- Good accuracy of stiffness scale

- Lower reconstruction error
- Underestimated stiffness
- ➢ Dominant Data-driven reconstruction!

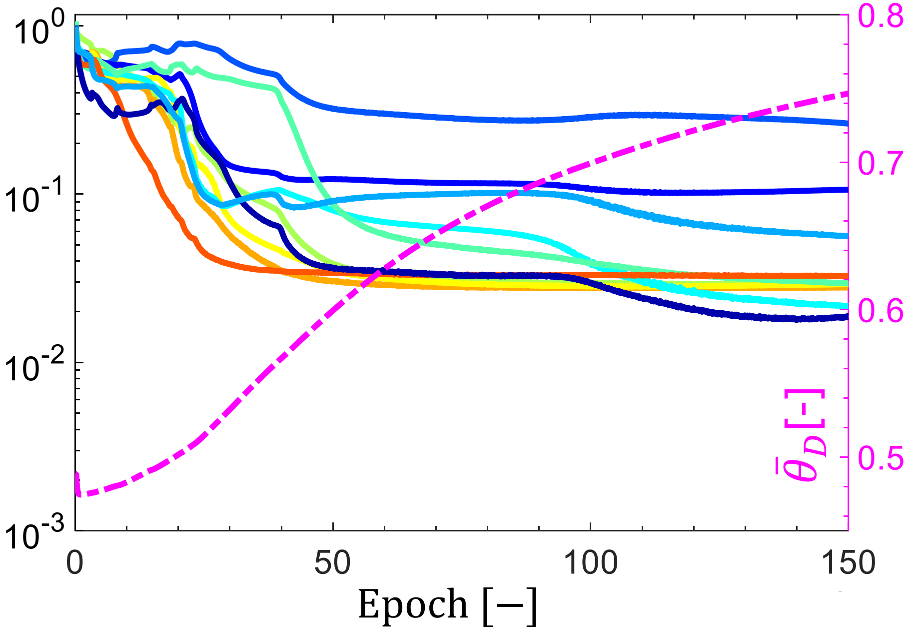
Evaluate k-PINN: Physics-informed Training

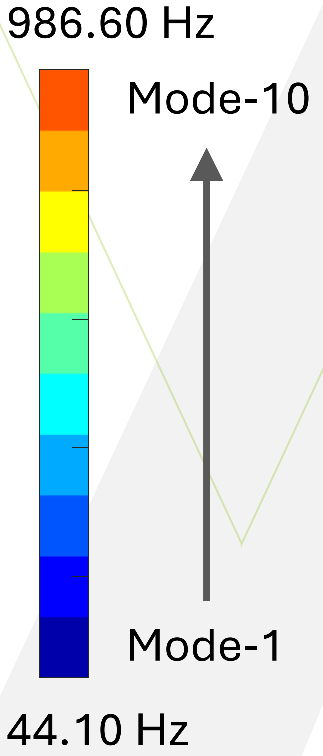
- Randomly Distributed Dataset of 96 Points

With Gaussian Mask



Without Gaussian Mask

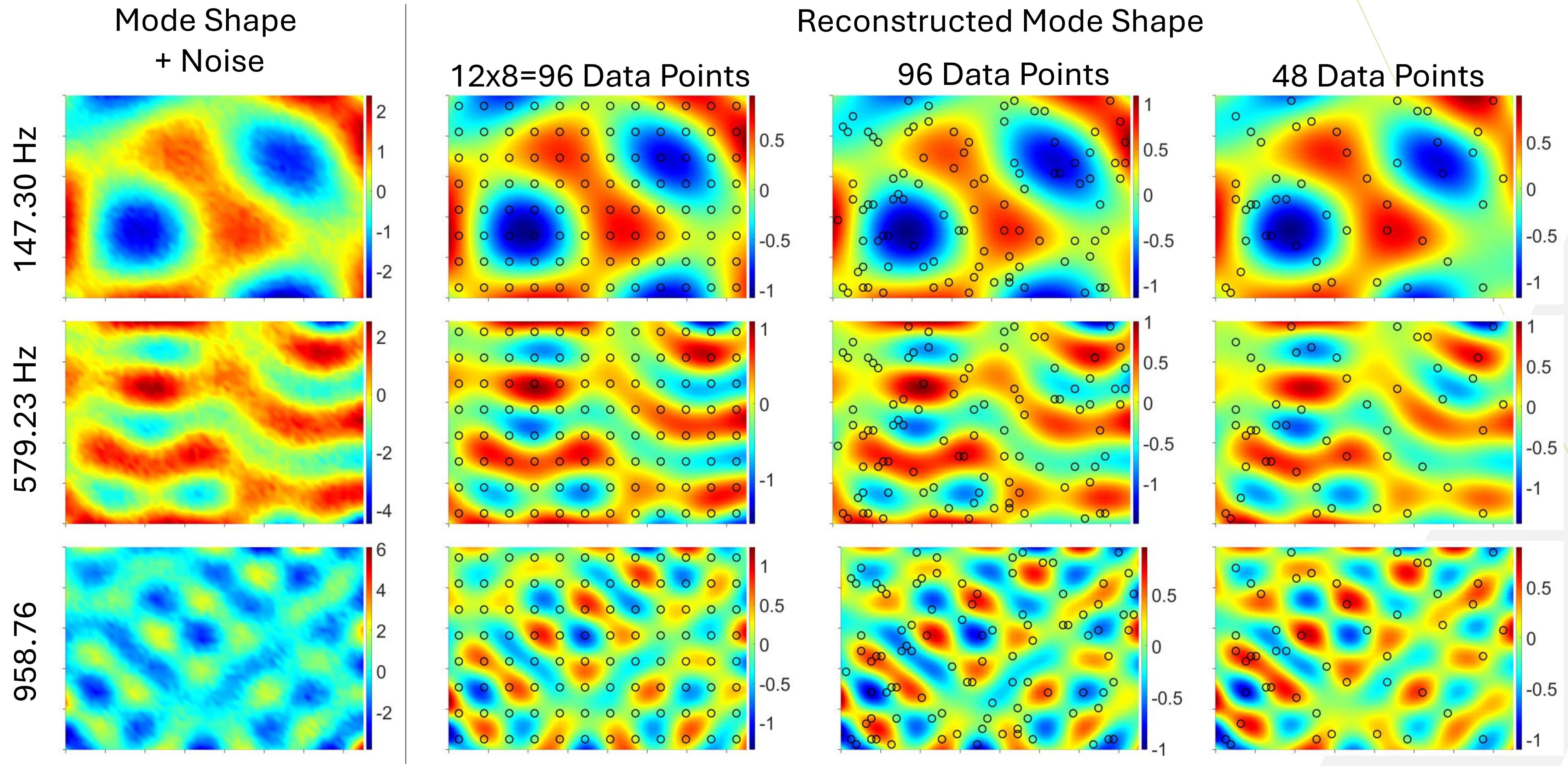




- Faster convergence by Gaussian Mask
- Good accuracy of stiffness scale
- Gaussian mask critical!

- Very high reconstruction error
- Underestimated stiffness
- ➢ Inefficient learning

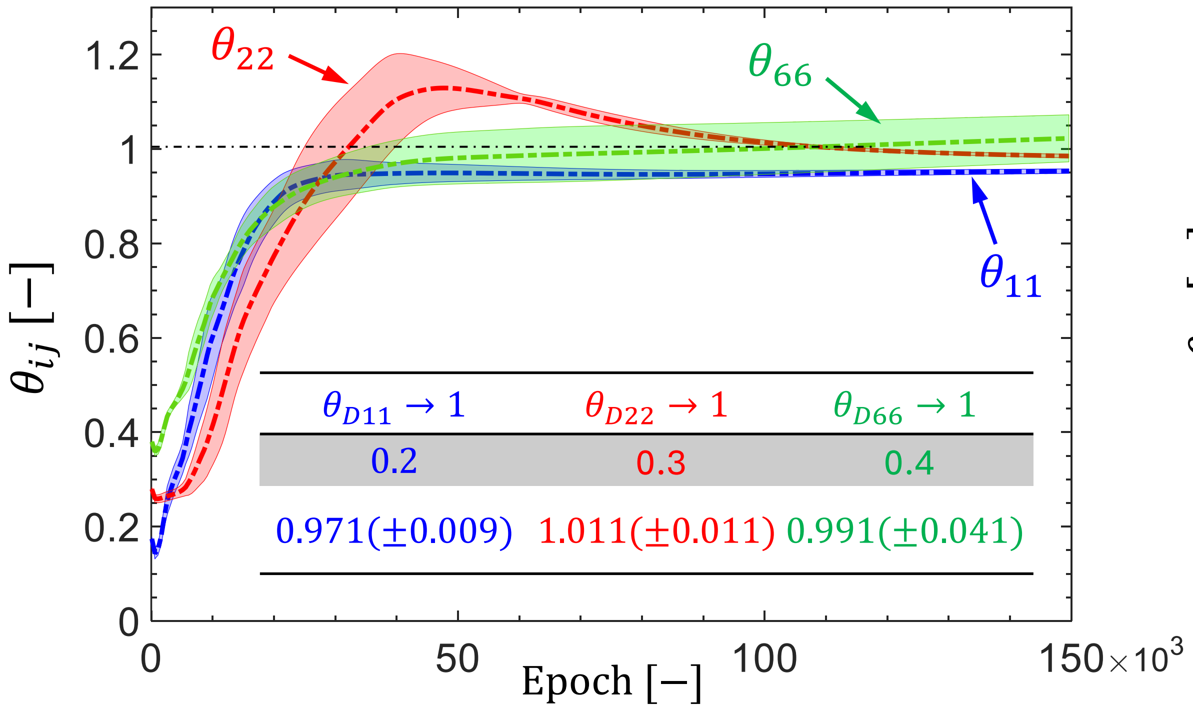
Full-field Reconstruction (Free bottom-right corner)



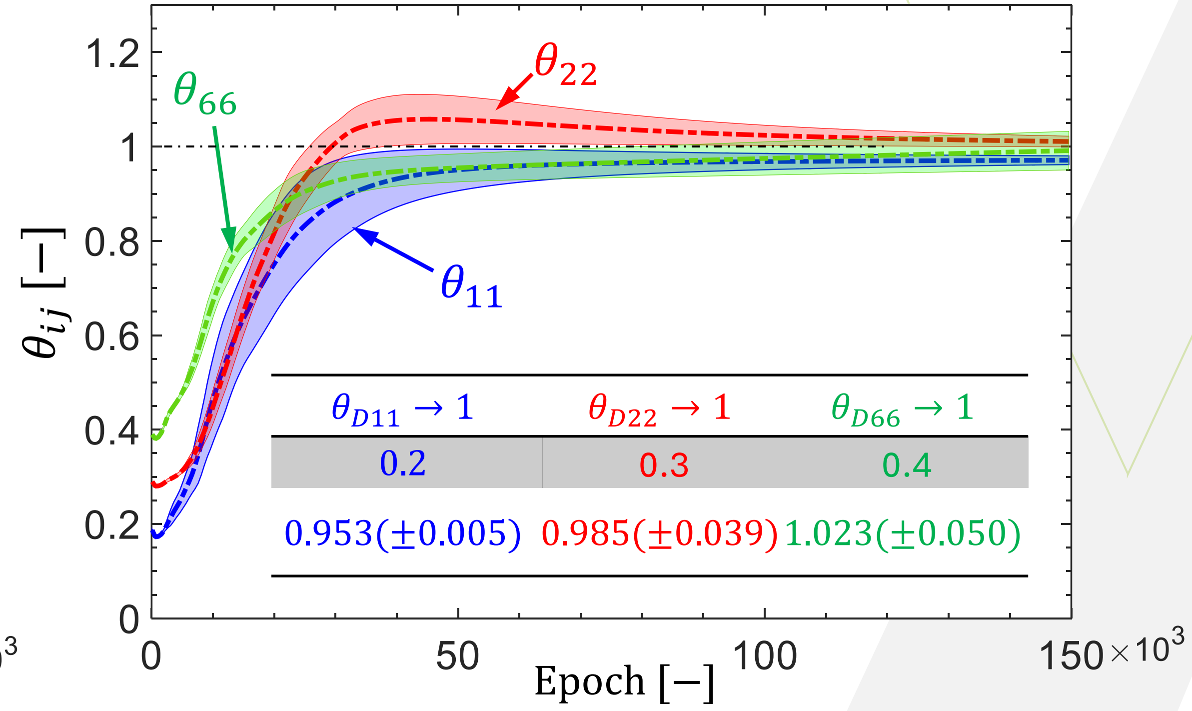
Identify Individual Stiffness Coefficients

- Uniformly distributed grid of 12x8=96 Data Points
- 10 Mode shapes

Symmetric BCs



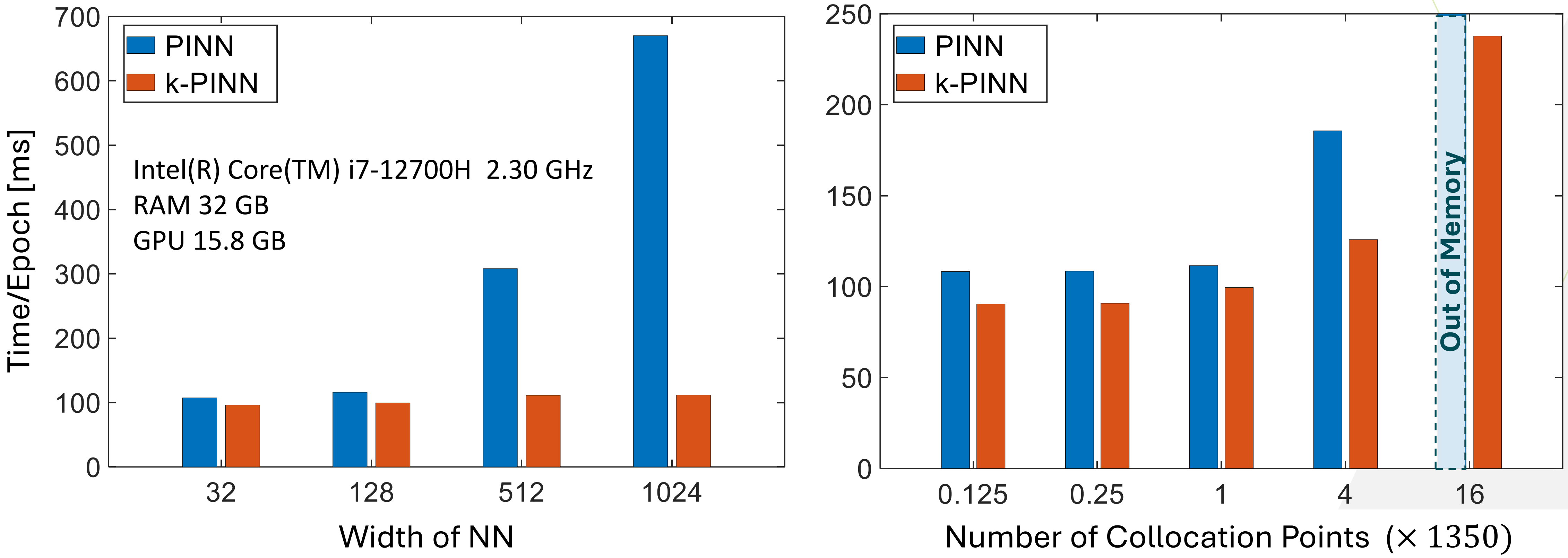
Asymmetric BCs



$$D = \{\boldsymbol{\theta_{D11}} D_{11}^0 \quad \boldsymbol{\theta_{D22}} D_{22}^0 \quad \boldsymbol{\theta_{D12}} D_{12}^0 \quad \boldsymbol{\theta_{D66}} D_{66}^0\}$$

Computational Efficiency

- Compute response and derivatives of any order in one forward solution
- Map to any number of collocation points, without involving NN
- Uniform grid of data and collocation points ➔ Fast Fourier Transform!



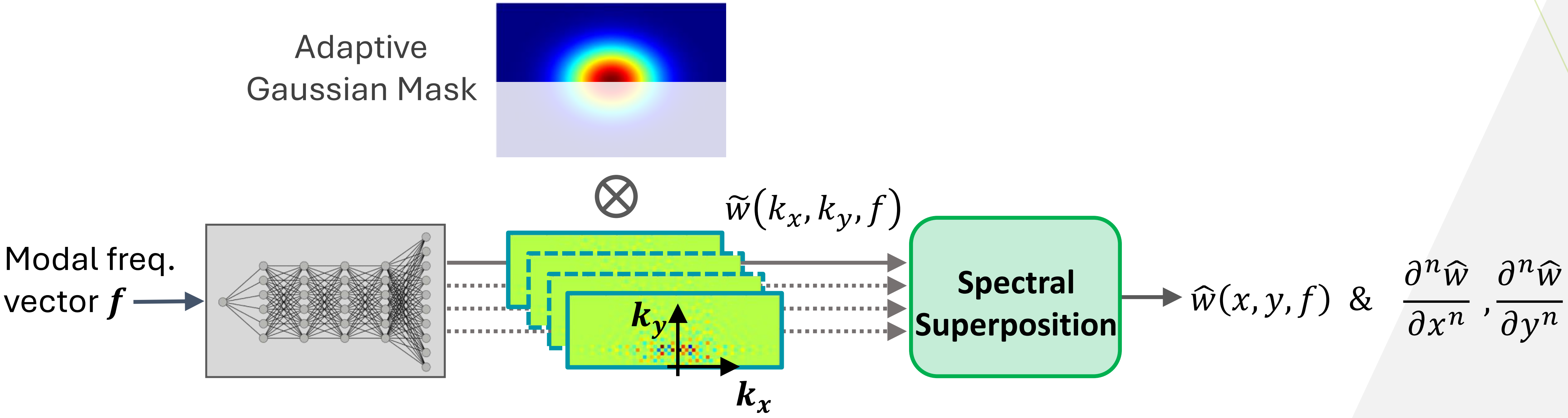
Concluding Remarks

k-space PINNs: Compressed Spectral solution with Adaptive Bandwidth

✓Tackle Spectral Bias ➔ Broadband Multimodal Reconstruction

✓Explainable Solution ➔ Symmetry & Wavenumber

✓Spectral Mapping and Derivatives ➔ Computational Cost



More details:

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"*k-space Physics-informed Neural Network (k-PINN) for Compressed Spectral Mapping and Efficient Inversion of Vibrations in Thin Composite Laminates*."
arXiv preprint arXiv:2404.03966 (2024).

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DRIVING INNOVATION IN MANUFACTURING

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